2/28/2001

Electro-Optic Modulation

$$\Delta(\epsilon_0 \chi_{ij}) = r_{ijk} E_k$$

r_{ijk} — linear electro-optic tensor

$$\overline{\overline{\chi}} = \chi_{ij} + \Delta\chi_{ij} = \chi_{ij} + \frac{1}{\epsilon_0} r_{ijk} E_k$$

E_k — application of external DC field

Different symmetry constrain various elements of the tensor.

KDB coordinates...

$$\chi_{\alpha\beta} = T_{\alpha i}\, \chi_{ij}\, T^{-1}_{j\beta} = T_{\alpha i} T_{\beta j} \chi_{ij}$$

$$= T_{\alpha i} T_{\beta j} \chi_{ij} + T_{\alpha i} T_{\beta j} T_{\gamma k} \epsilon_0^{-1} r_{ijk} \underbrace{T_{\gamma l} E_l}_{E_\gamma}$$

most general transformation

easy on computer

Methods of Solution

I. • Consider the effect of external field on χ using r_{ijk} or r_{Ik}

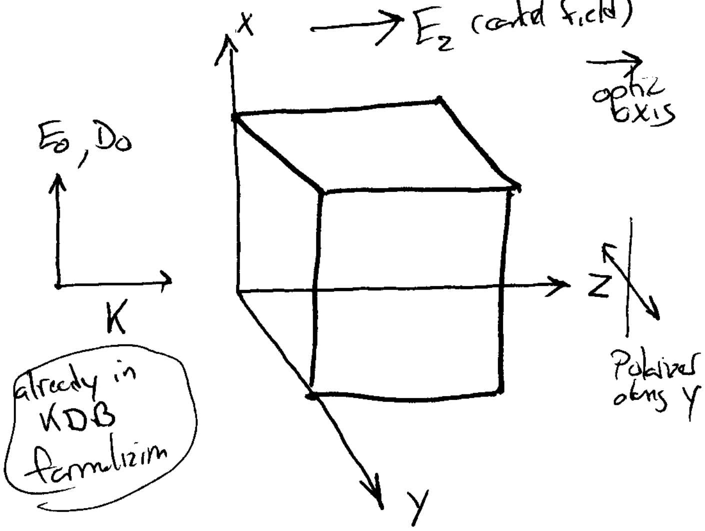
• Solve KDB Maxwell's Equations.

II. • Consider the effect of external field on the index ellipsoid.

• Solve using geometric model.

Ex KDP potassium dihydrogen phosphate

Modulator (longitudinal Geometry)



$$\overline{\overline{\chi}} = \begin{bmatrix} \chi & \frac{1}{\epsilon_0} r_{63} E_z^c & 0 \\ \frac{1}{\epsilon_0} r_{63} E_z^c & \chi & 0 \\ 0 & 0 & \chi_z \end{bmatrix}$$

only nonzero here, cuz of the direction along which we've applied the control field.

• Already in KDB system because the wave propagates along $\hat{z}$.

So since in KDB

$$\begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0 & \omega/k \\ -\omega/k & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\begin{bmatrix} \nu & 0 \\ 0 & \nu \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & -\omega/k \\ \omega/k & 0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

cross eliminate terms:

$$\begin{bmatrix} u^2-\gamma\chi & -\gamma\Delta\chi \\ -\gamma\Delta\chi & u^2-\gamma\chi \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = 0$$

$$\Delta\chi = \frac{1}{\epsilon_0} r_{63} E_z^c = $$

$$= \chi_0 r_{63} E_z^c$$

Find Eigenvalues & Vectors

Det=0

$$(u^2-\gamma\chi)^2 - (\gamma\Delta\chi)^2 = 0$$

$$u^4 - 2\gamma\chi u^2 + (\gamma\chi)^2 - (\gamma\Delta\chi)^2 = 0$$

$$u^2 = \frac{1}{2}\left[2\gamma\chi \pm \sqrt{4(\gamma\chi)^2 - 4((\gamma\chi)^2 - (\gamma\Delta\chi)^2)}\right]$$

$$u^2 = \gamma\chi \pm \gamma\Delta\chi$$ - two characteristic phase velocities

$$u_\pm^2 = \gamma\chi \pm \gamma\Delta\chi$$

recall

$u = \frac{\omega}{k}$ usually we want k

$$k = \omega/u$$

$$k_\pm = \frac{\omega}{u_\pm} = \omega\left[\gamma\chi \pm \gamma\Delta\chi\right]^{-1/2}$$

$$= \frac{\omega}{\sqrt{\gamma\chi}}\left[1 \pm \frac{\Delta\chi}{\chi}\right]^{-1/2}$$

power series expand

$$k_\pm \approx \frac{\omega}{\sqrt{\gamma\chi}}\left[1 \mp \frac{1}{2}\frac{\Delta\chi}{\chi}\right]$$

$$k_\pm \approx \frac{\omega}{\sqrt{\gamma\chi}} \mp \frac{\omega\gamma\Delta\chi}{2(\gamma\chi)^{3/2}}$$ So This is the wave vector

recall

$$\frac{n_o}{c} = (\gamma\chi)^{-1/2}$$

so

$$k_\pm \approx \frac{\omega}{c} n_o \mp \frac{1}{2}\omega\frac{n_o^3}{c^3}\gamma\chi_o r_{63} E_z^c$$

$$\boxed{k_\pm \approx \frac{\omega}{c} n_o \mp \frac{1}{2} n_o^3 r_{63} E_z^c}$$

Now Eigenvectors... (which are allowed polarizations) substitute back into

we

- $u_+^2 = \gamma\chi + \gamma\Delta\chi \Rightarrow D_1 = -D_2$

$$\Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

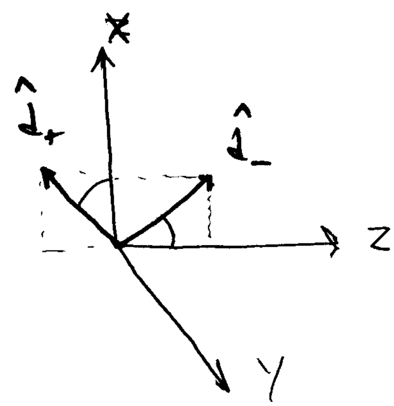
so allowed polarization is

$$\hat{d}_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- $u_-^2 = \gamma\chi - \gamma\Delta\chi$

$D_1 = D_2$

$$\hat{d}_- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Incident wave is $\bar{D}^{in} = \hat{X} D^{in}$

Excite both $\hat{d}_+$ and $\hat{d}_-$ waves.

$$D_+ = \hat{d}_+ \cdot \bar{D}^{in} = \frac{1}{\sqrt{2}} D^{in}$$

$$D_- = \hat{d}_- \cdot \bar{D}^{in} = \frac{1}{\sqrt{2}} D^{in}$$

so input can be written

$$\bar{D}^{\omega} = \hat{d}_+ \frac{1}{\sqrt{2}} D^{\omega} + \hat{d}_- \frac{1}{\sqrt{2}} D^{\omega}$$

(incident radiation resolves into 2 waves)

The waves D_+ and D_- propagate with different velocities & K vectors. There is a phase shift between waves. After a length $\underline{\ell}$ of propagation

$$\bar{D}_+ = D_+ e^{iK_+\ell} \hat{d}_+$$

$$\bar{D}_- = D_- e^{-iK_-\ell} \hat{d}_-$$

After the modulator we project out the $\hat{y}$ component of waves.

$$\bar{D}^{out} = \hat{y} \cdot \bar{D} = \hat{y} \cdot (\bar{D}_+ + \bar{D}_-)$$

$$= \left(D_+ e^{iK_+\ell} \hat{y} \cdot \hat{d}_+ + D_- e^{iK_-\ell} \hat{y} \cdot \hat{d}_-\right)$$

$$= \frac{1}{\sqrt{2}}\left(-D_+ e^{iK_+\ell} + D_- e^{iK_-\ell}\right)$$

$$= \frac{1}{2} D^{in}\left(e^{-iK_+\ell} + e^{iK_-\ell}\right)$$

$$\frac{D^{out}}{D^{in}} = e^{i(K_+ + K_-)\ell/2} \frac{1}{2} \times \left[e^{i(K_- - K_+)\ell/2} - e^{-i(K_- - K_+)\ell/2}\right]$$

so

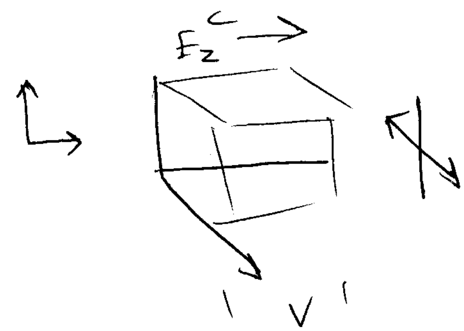
$$\frac{I^{out}}{I^{in}} = \sin^2 (K_- - K_+) \frac{\ell}{2}$$

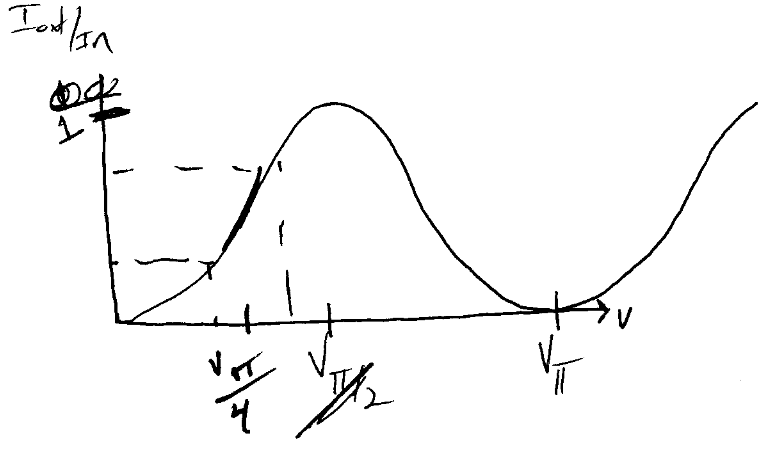
$$= \sin^2\left[\frac{1}{2} \frac{\omega}{c} n_0^3 r_{63} E_z^c \ell\right]$$

relate to applied voltage.

$$\frac{I_{out}}{I_{in}} = \sin^2\left[\pi V / V_\pi\right]$$

$$V_\pi = \frac{\lambda}{n_0^3 r_{63}}$$





bias to a linear region

For KDP

$r_{41} = 8.6 * 10^{-12}$ m/V

$r_{63} = 10.6 * 10^{-12}$ m/V

uniaxial $n_o = 1.51$

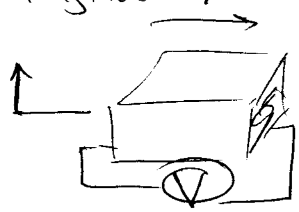
$n_e = 1.47$

$n_o^3 r_{63} = 36 * 10^{-12}$ m/V

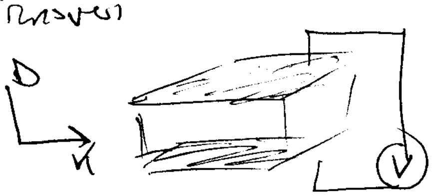
$\lambda = .5\ \mu m$

$V_{\frac{\pi}{2}} \approx 7.5$ KVolts

we did longitudinal



also Transverse



hmwk

Indicatrix Method

Ex KDP modulator (longitudinal)

$$\Delta(\epsilon_o \chi_{ij}) = r_{ijk} E_k$$

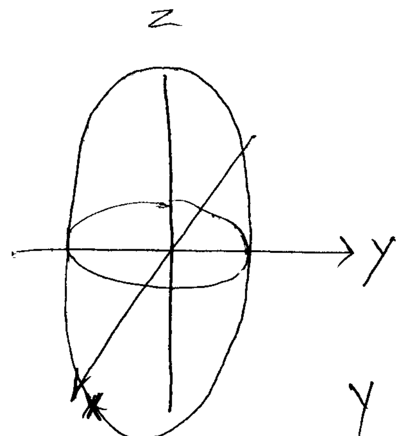
many book use this approach

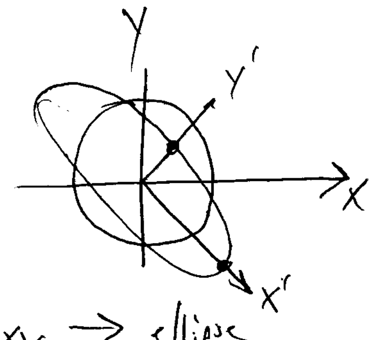
Indicatrix:

$$(\epsilon \chi_{ij} + r_{ijk}) E_k x_i x_j = 1$$

For this example

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2 r_{63} E_z^c xy = 1$$





cross terms in xy → ellipse

$$x = \frac{1}{\sqrt{2}}(x' + y')$$
$$y = \frac{1}{\sqrt{2}}(-x' + y')$$

This transformation diagonalizes this form.

principal axis transformation

$$x^2 = \frac{1}{2}(x'^2 + 2x'y' + y'^2)$$
$$y^2 = \frac{1}{2}(y'^2 - 2x'y' + y'^2)$$

$$xy = \frac{1}{2}(-x'^2 + y'^2)$$

and we get

$$\left(\frac{1}{n_o^2} - r_{63} E_z^c\right) x'^2 + \left(\frac{1}{n_o^2} + r_{63} E_z^c\right) y'^2 + \frac{z^2}{n_e^2} = 1$$

Now what are the intercepts?

$$\left(\frac{1}{n'_{xx}}\right)^2 = \frac{1}{n_o^2} - r_{63} E_z^c$$

$$\left(\frac{1}{n'_{yy}}\right)^2 = \frac{1}{n_o^2} + r_{63} E_z^c$$

$$\left(\frac{1}{n_{xy}'^2}\right) = \left(\frac{1}{n_o^2}\right)\left[1 - n_o^2 r_{63} E_z^c\right]$$

$$n_{xx}' = n_o\left[1 - n_o^2 r_{63} E_z^c\right]^{-1/2}$$

$$\approx n_o\left[1\right.$$

$$\approx n_o + n_o^3 \frac{1}{2} r_{63} E_z^2$$

$$\approx n_o + n_o^3 \frac{r_{63}}{2} E_z^2$$

$$n_{yy} \approx n_o - \frac{n_o^3 r_{63} E_z^2}{2}$$

characteristic waves polarized along
x' & y' $\quad D_x'$ & D_y'

$D_x' e^{i\frac{w}{c} n_{xx}' l}$

$D_y e^{i w/c\, n_{yy}' l}$ } in end get same result.

3/7/2001

Finished w/ EO modulator

Now onto something more non-linear.

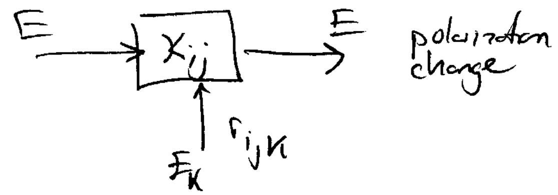
control of optical fields:

$E e^{jkz}$ (phase)

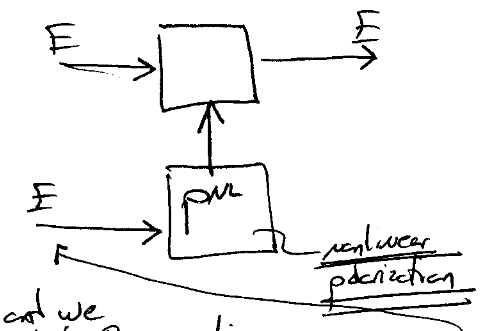
(amplitude)

either amp or phase vs (V or i)

EO Modulation (dc control field)



optical Analog (optical field)



can we control the nonlinear polarization w/ another optical field.

use Eq for $P^{NL} \rightarrow$ Energy Transfer

$$\nabla \times E = -\frac{\partial}{\partial t}\mu_0 H$$

$$\nabla \times H = J + \frac{\partial}{\partial t} D; \quad D = \epsilon E + P^{NL}$$

$$= J + \frac{\partial}{\partial t}\epsilon E + \frac{\partial}{\partial t}P^{NL}$$

lets look at energy density

Poynting - Energy conservation law

$$(\nabla \times E)\cdot H = -\left(\frac{\partial}{\partial t}\mu_0 H\right)\cdot H$$

$$= -\frac{1}{2}\frac{\partial}{\partial t}(\mu_0 H^2)$$

$$E\cdot(\nabla\times H) = E\cdot J + E\left(\frac{\partial}{\partial t}\epsilon E\right) + E\frac{\partial}{\partial t}P^{NL}$$

$$= E\cdot J + \frac{1}{2}\frac{\partial}{\partial t}(E\cdot\epsilon\cdot E) + E\frac{\partial}{\partial t}P^{NL}$$

Quad terms ~~...~~ are energy densities

Electric Energy Density $w_e = \frac{1}{2} E\cdot\epsilon\cdot E$

Magnetic $w_m = \frac{1}{2}\mu_0 H\cdot H$

note

$$\nabla\cdot(E\times H) = (\nabla\times E)\cdot H - (\nabla\times H)\cdot E$$

so

$$\nabla\cdot(E\times H) + \dot{w}_e + \dot{w}_m + \underbrace{E\cdot J}_{\text{dissipative}} + E\cdot\frac{\partial}{\partial t}P^{NL} = 0$$

energy transfer from polarization into field.

$$\langle u\rangle = -\left\langle E\cdot\frac{\partial}{\partial t}P^{NL}\right\rangle$$

$\Rightarrow$ energy transfer from P^{NL} into fields.

In complex form ($e^{j\omega t}$)

$$\langle u\rangle = -\frac{1}{2}\mathrm{Re}\{ j\omega E\cdot P^{NL*}\}$$

$$= \frac{1}{2}\mathrm{Im}\{\omega E^{*}\cdot P^{NL}\}$$

Depending on phase of nonlinear polarization relative to E, can transfer energy into ~~or~~ out of the fields.

control of phase of P^{NL} relative to E is critical!!

so how do we control it?

Wave Eq.

$$\nabla\times E = -\frac{\partial}{\partial t}\mu_0 H$$

$$\nabla\times H = J + \frac{\partial}{\partial t}\epsilon\cdot E + \frac{\partial}{\partial t}P^{NL}$$

σE

cross diminate one of fields

$$\nabla\times\nabla\times E = -\frac{\partial}{\partial t}\mu_0\nabla\times H$$

$$= -\mu_0\frac{\partial}{\partial t}\sigma E - \mu_0\frac{\partial^2}{\partial t^2}\epsilon E - \mu_0\frac{\partial^2}{\partial t^2}P^{NL}$$

$$\nabla\times\nabla\times E + \mu_0\frac{\partial}{\partial t}\sigma E + \mu_0\epsilon\frac{\partial^2}{\partial t^2}E = \underline{-\mu_0\frac{\partial^2}{\partial t^2}P^{NL}}$$

looks like source term to wave eq.

The nonlinear polarization acts like a source term.

consider Time Harmonic $e^{j\omega t}$ $\frac{\partial}{\partial t}\to j\omega$

$$(\nabla\times\nabla\times + j\omega\mu_0\sigma - \omega^2\mu_0\epsilon)E = \omega^2\mu_0 P^{NL}$$

Normally to simplify consider no source terms

$$\nabla\times\nabla\times E = \nabla(\nabla\cdot E) - \nabla^2 E$$

usually assume $\nabla\cdot D = 0 \to \nabla\cdot E = 0$

if tensor material This is not true!!

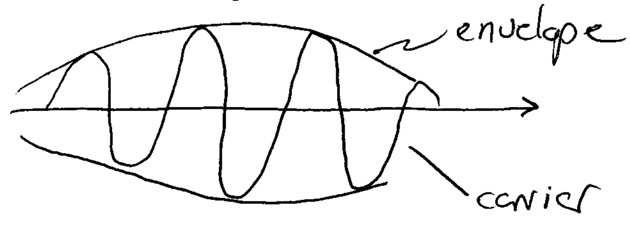
But still for most cases is a reasonable approximation

so

$$\left(\nabla^2 - \mu_0\sigma\frac{\partial}{\partial t} - \mu_0\epsilon\frac{\partial^2}{\partial t^2}\right)E = \mu_0\frac{\partial^2}{\partial t^2}P^{NL}$$

Assume

Slowly varying envelope approximation



$$E(z,\omega,t) = \hat{e}\,\underbrace{E(z,t)}_{\text{envelope}}\,\underbrace{e^{-j(kz-\omega t)}}_{\text{carrier/wave}}$$

Same thing for Non-linear polarization:

$$P^{NL}(z,\omega,t) = \hat{p}\,\underbrace{P^{NL}(z,t)}_{\text{envelope}}\,\underbrace{e^{j(k_p z-\omega t)}}_{\text{carrier}}$$

now go to wave equation to see what This implies

But first we need derivatives:

$$\frac{\partial}{\partial t}E \Rightarrow \left[\frac{\partial}{\partial t} + j\omega\right]E\,e^{j(kz-\omega t)}$$

assume only operate on envelope

$$\frac{\partial^2}{\partial t^2}E \Rightarrow \underbrace{\left[\frac{\partial^2}{\partial t^2} + 2j\omega\frac{\partial}{\partial t} - \omega^2\right]}_{\text{only on envelope}}E\,e^{j(kz-\omega t)}$$

$$\frac{\partial^2}{\partial z^2}E \Rightarrow \left[\frac{\partial^2}{\partial z^2} - 2jk\frac{\partial}{\partial z} - k^2\right]E\,e^{j(kz-\omega t)}$$

$$\frac{\partial^2}{\partial t^2}P^{NL} \Rightarrow \left[\frac{\partial^2}{\partial t^2} + 2j\omega\frac{\partial}{\partial t} - \omega^2\right]P^{NL}e^{j(k_p z-\omega t)}$$

- substitute into wave equation
$\Rightarrow$ eq for envelope functions

$$\left\{\left(\cancel{\frac{\partial^2}{\partial z^2}}-2jk\frac{\partial}{\partial z}-\cancel{k^2}\right)-\mu_0\sigma\left(\frac{\partial}{\partial t}+jw\right)-\mu_0\epsilon\left(\cancel{\frac{\partial^2}{\partial t^2}}+2jw\frac{\partial}{\partial t}-\cancel{w^2}\right)\right\}E\,e^{j(kz-wt)}$$

$$=\mu_0\left(\cancel{\frac{\partial^2}{\partial t^2}}+\cancel{2jw\frac{\partial}{\partial t}}-w^2\right)\hat{e}\cdot\hat{p}\,P^{NL}e^{j(k_pz-wt)}$$

(2) small wrt; (1); (2) wrt; (1); (2); (2) wrt to

Try to get equation for envelope

How to simplify This?

(1) Recall dispersion relation $k^2=\mu_0\epsilon w^2$

(2) slowly varying envelope approximation
(compared to time variation & compared to space variation of carrier)

- $k\frac{\partial E}{\partial z} >> \frac{\partial^2 E}{\partial z^2}$ neglect curvature wrt first derivative

or
ΔE negligible for $\Delta z < \lambda$

- $w\frac{\partial E}{\partial z} >> \frac{\partial^2 E}{\partial t^2}$

or
ΔE negligible for $\Delta t < 1/w$

for polarization:
$$w^2P >> w\frac{\partial}{\partial t}P >> \frac{\partial^2}{\partial t^2}P$$

so now

$$\left[-2jk\frac{\partial}{\partial z}-jw\mu_0\sigma-2jw\mu_0\epsilon\frac{\partial}{\partial t}\right]E\,e^{j(kz-wt)}=-\mu_0w^2(\hat{e}\cdot\hat{p})P^{NL}e^{j(k_pz-wt)}$$

$$\left[\frac{\partial}{\partial z}+\frac{\mu_0\sigma}{2}\frac{w}{k}+\mu_0\epsilon\frac{w}{k}\frac{\partial}{\partial t}\right]E=\frac{-j\mu_0w^2}{2k}(\hat{e}\cdot\hat{p})P^{NL}e^{j(k-k_p)z}$$

Recall
$$\frac{w}{k}=\frac{c}{n}\qquad\left(\frac{n}{c}\right)^2=\mu_0\epsilon$$

define
$$\alpha\equiv\frac{\mu_0\sigma c}{2n}\quad\text{(loss coefficient)}$$

$$\boxed{\left[\frac{\partial}{\partial z}E+\alpha E+\frac{n}{c}\frac{\partial}{\partial t}E\right]=\frac{-j\,w\mu_0c}{2n}(\hat{e}\cdot\hat{p})P^{NL}e^{j(k-k_p)z}}$$

Steady state $\frac{\partial}{\partial t}\to 0$ $\quad\sigma=0$

$$\boxed{\frac{\partial}{\partial z}E=\frac{-j\,w\mu_0c}{2n}(\hat{e}\cdot\hat{p})P^{NL}e^{j(k-k_p)z}}$$

Spatial Evolution of Nonlinear envelope

If $\frac{\partial}{\partial z}E$ is real (imaginary P^{NL})
Then we have growth or decay of E envelope.

If $\frac{\partial}{\partial z}E$ is imaginary (real P^{NL} i.e. in phase w/ field)

Then we have phase shift

Now what do we do with This?

Simplest example

Second Harmonic Generation:

$P^{(2)} \sim \underline{EE}$ Tensor & bunch of other stuff

Describe This by an "effective" nonlinear coefficient which is related to the susceptibility $\chi^{(2)}$

$$P(\omega_2) = \epsilon_0 \, d_{eff}(\omega_2 : \omega_1, \omega_1) \, E(\omega_1) E(\omega_1)$$

$\downarrow K_p$ $\quad$ $\downarrow K(\omega_1)$ $\quad \omega_2 = 2\omega_1$

i.e generate a nonlinear polarization proportional to ~~the~~ square of field.

$$K_p = K(\omega_1) + K(\omega_1) = 2K(\omega_1)$$

$$\frac{\partial}{\partial z} E(\omega_2) = \frac{-j \omega_2 \mu_0 c}{n_2} \epsilon_0 d_{eff} E(\omega_1) E(\omega_1) e^{j(K(\omega_2) - 2K(\omega_1))z}$$

$$\boxed{\frac{\partial}{\partial z} E(\omega_2) = \frac{j \omega_2}{n_2 c} d_{eff} E(\omega_1) E(\omega_1) e^{j \Delta K z}}$$

actually This is one equation in a set of nonlinear equation other would be $\frac{\partial}{\partial z} E(\omega_1) = \sim E(\omega_1) E(\omega_2)$

define

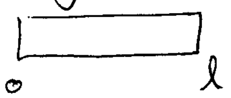
$$\Delta K \equiv \underbrace{K(\omega_2)}_{\substack{= \\ 2^{nd} \text{ Harmonic} \\ K}} - \underbrace{2K(\omega_1)}_{\substack{= \\ K_p \text{ of nonlinear} \\ \text{polarization}}} \equiv \underline{\text{phase mismatch}}$$

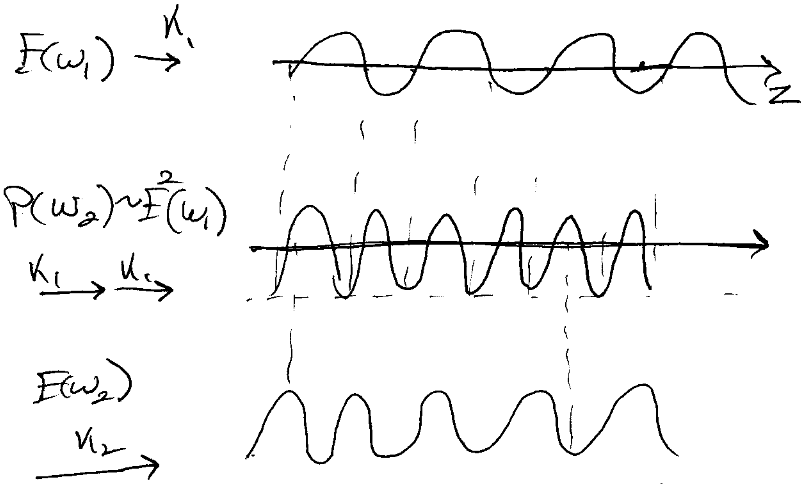
Approximation – low conversion (linear)

i.e. $E(\omega_2) << E(\omega_1)$

$E(\omega_1)$ approximately constant.

so can now integrate directly from 0 to l





b/c isn't necessarily phase matching

$$E(\omega_2, z=l) = \frac{-j\omega_2 d_{eff}}{n_2 c} E^2(\omega_1) \int_0^l e^{j\Delta k z}\, dz$$

⋮

$$= \frac{-j\omega_2 d_{eff}}{n_2 c} E^2(\omega_1)\, l \left[\frac{\sin \frac{\Delta k l}{2}}{\Delta k l/2}\right] e^{j\frac{\Delta k l}{2}}$$

Calculate the Intensity

$$I = \frac{1}{2} n \sqrt{\frac{\epsilon_0}{\mu_0}} |E|^2$$

so

$$I(\omega_2, z=l) = \frac{2\omega_2^2 d_{eff}^2}{n_2^3 c^3 \epsilon_0} l^2 I^2(\omega_1) \cdot \left[\frac{\sin \Delta k l/2}{\Delta k l/2}\right]^2$$

SH (2nd Harmonic) intensity $I(2\omega)$ depends on

l^2 and $[\sin \Delta k l/2 \,/\, \Delta k l/2]^2$

Assume phase matching $\Delta k = 0$

$$I(\omega_2, l) = \frac{2\omega_2^2 d_{eff}^2}{n_2^3 c^3 \epsilon_0} l^2 I^2(\omega_1)$$

or

$$\frac{I(\omega_2, l)}{I(\omega_1)} = \Gamma^2 l^2$$

$$\Gamma^2 = \frac{2\omega_2^2 d_{eff}^2}{n_2^3 c^3 \epsilon_0} I(\omega_1)$$

$$\Gamma^2 = \frac{\omega_2^2 d_{eff}^2}{n_2 c^2} |E|^2$$

so somehow we need to set $\Delta k = 0$

how to control phase?

Slowly varying envelope equation for P^{NL}

$$\frac{\partial}{\partial z}E + \alpha E + \frac{n}{c}\frac{\partial}{\partial t}E = \frac{-j\omega\mu_0 c}{2n}(\hat{e}\cdot\hat{p})\, P^{NL}\, e^{j(K-K_p)z}$$

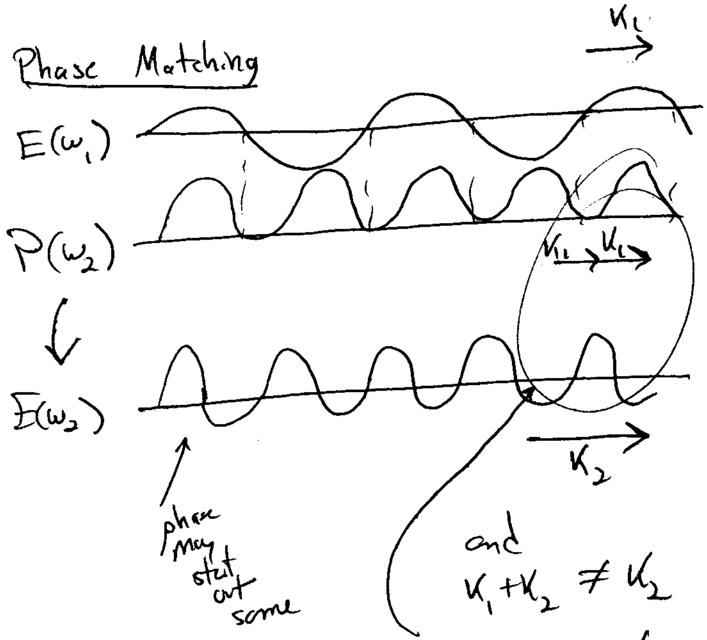
2nd Harmonic generation is

$$I(\omega_2, z=\ell) = \frac{2\omega_2^2 d^2}{n_2^3 c^3 \epsilon_0}\, \ell^2\, I^2(\omega_1)\left[\frac{\sin \Delta K\ell/2}{\Delta K\ell/2}\right]^2$$

which started from

$$P(\omega_2) = \epsilon_0 d\, E(\omega_1)E(\omega_1)$$

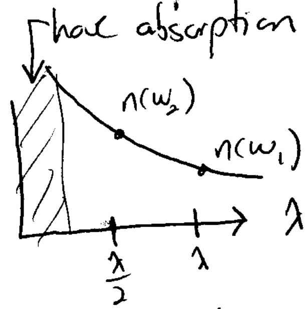
Phase Matching



so spatially out of phase!

so How to do phase matching?

Materials: most optical materials have absorption in UV



Have to set up situation where we make the phases the same $\Longleftrightarrow$ make the indices equivalent

$I(\omega_2) \to 0$ when $\Delta K\ell = \pi$

There is a characteristic length, the coherence length ℓ_c, over which fields go out of phase.

so we only get interaction if the length is much shorter than the coherence length.

$$\ell_c = \pi/\Delta K = \lambda_\omega / 4(n(\omega_2) - n(\omega_1))$$

Example GaAs w/ λ in IR (1.5μ range)

$\ell_c = 10 - 100\,\mu m$ → short crystal

How do we overcome this?

so we have

$$P(\omega_2) = \epsilon_0 d_{eff}(\omega_2 : \omega_1\, \omega_1)\, E(\omega_1)E(\omega_1)$$

$$\Delta K = K(\omega_2) - K(\omega_1) - K(\omega_1)$$

To have phase matching set $\Delta K = 0$ or alternatively set $n(\omega_2) = n(\omega_1)$ $\quad \omega_2 = 2\omega_1$

How do we do this?

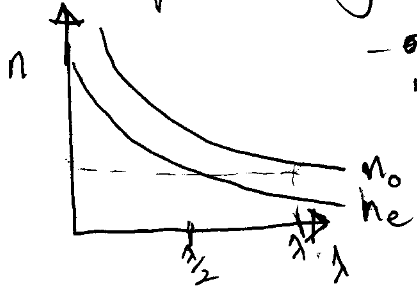
We know anisotropic materials different waves have different velocities

we can use anisotropic properties of $\bar{\epsilon}$ to perform phase matching.

Uniaxial material → ordinary & extraordinary waves.

can we use these different waves to match the fundamental & 2nd harmonic.

Ex 90° phase matching



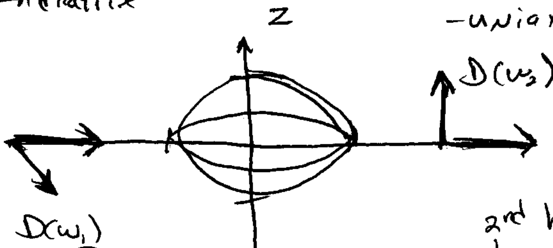
- uniaxial material ext & ord wave

or

$$n_o(\omega_1) = n_e(\omega_2)$$

True for only a certain $\omega_2 = 2\omega_1$
(for a given crystal only works for one wavelength)

Indiatrix



-uniaxial

$D(\omega_1)$
out of paper
no projection into z
ordinary wave.

2nd harmonic along the optic axis

$D^o(\omega_1)$ is polarized $\perp$ to optic axis $\hat{z}$

$D^e(\omega_2)$ is generated, polarized $\parallel$ to optic axis. $\hat{z}$.

The optic axis is 90° to the propagation.

Note in this case the Poynting vector is parallel to the propagation axis.

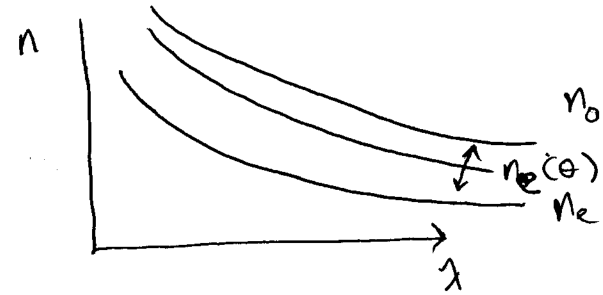
Note: This works for only one wavelength.

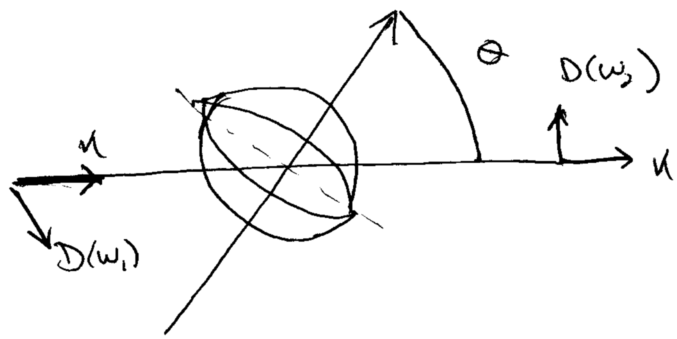
How to generalize?

~~[illegible]~~

Type I Phase Matching

Change crystal orientation





so $n_e(\theta)$ can be tuned to be someplace between n_o & n_e

so for a give λ we can find ~~the~~ an angle so that

$$n_e(\theta, \lambda/2) = n_o(\lambda)$$

i.e

$$n_e(\omega_2, \theta) = n_o(\omega_1)$$

find θ

$D^o(\omega_1)$ polarized $\perp$ to optic axis $\hat{z}$.

$D^e(\omega_2)$ has component along optic axis.

Recall

ordinary wave $\quad u = (\chi v)^{1/2}$

$n_o(\theta) = n_o$

$D = \hat{e}_1 D_1$

$H = \hat{e}_2 \dfrac{v\chi}{u} D_1$

$E = \hat{e}_1 \chi D_1$

$S =$ in $\hat{e}_3$ direction

uniaxial material

Extraordinary wave

$$u = [v\chi\cos^2\theta + v\chi_z\sin^2\theta]^{1/2}$$

$$n_e(\theta) = \left[\left(\frac{\cos\theta}{n_o}\right)^2 + \left(\frac{\sin\theta}{n_e}\right)^2\right]^{-1/2}$$

$D = \hat{e}_2 D_2$

$H = -\hat{e}_1 \dfrac{v}{u}[\chi\cos^2\theta + \chi_z\sin^2\theta]$

$E = \hat{e}_2 [\chi\cos^2\theta + \chi_z\sin^2\theta] D_2 + \hat{e}_3[(\chi - \chi_z)\sin\theta\cos\theta] D_2$

E×H - poynting is not in same direction as direction of propogation

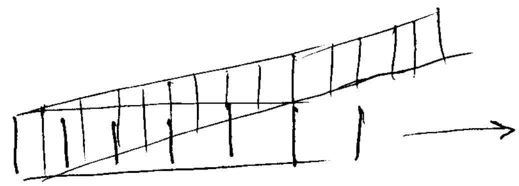
$$S = E\times H = \hat{e}_3 \frac{v}{u}[\chi\cos^2\theta + \chi_z\sin^2\theta]^2 |D_2|^2 + \hat{e}_2 \frac{v}{u}[\chi\cos^2\theta + \chi_z\sin^2\theta]^2 \cdot (\chi - \chi_z)\sin\theta\cos\theta\, |D_2|^2$$

S is not in the same direction as $\hat{e}_3$

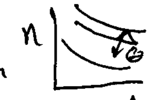
S is at angle ρ WRT $\hat{e}_3$

$$\tan\rho = |s_2|/|s_3| = \tfrac{1}{2}[n_e(\theta)]^2\left(\frac{1}{n_o^2} - \frac{1}{n_e^2}\right)\sin 2\theta$$

since poynting is at angle ρ 2nd Harmonic walks off at a different angle

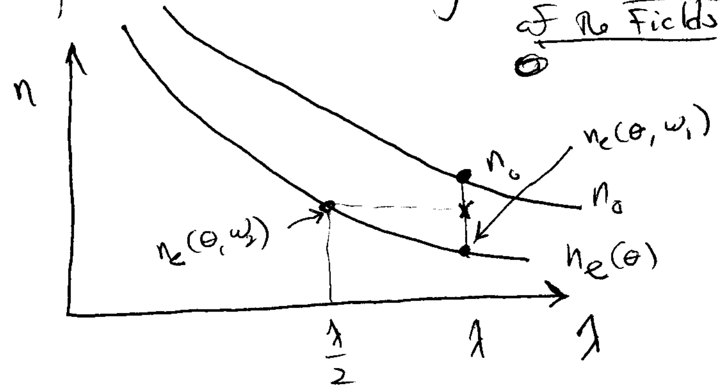


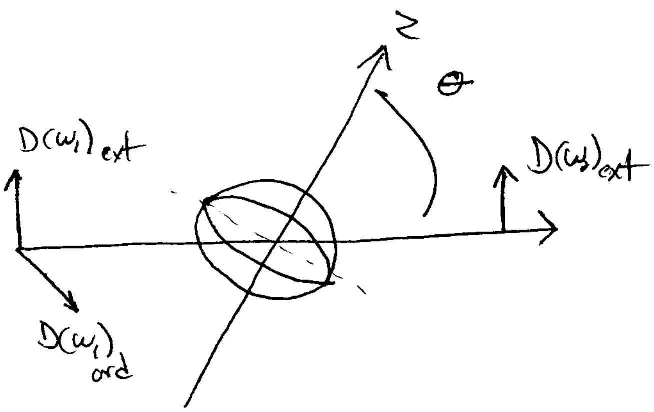
There is still a range restriction over the range that $n_e(\theta)$ varies & we only have a certain length before ω_2 wave walks off.



Type II phase matching

combination of ord & extr for at least one of the fields





$\Delta k = k(\omega_2) - k(\omega_1) - k(\omega_1) \to 0$

so chose angle to satisfy

$$n_e(\theta,\omega_2) = \tfrac{1}{2}[n_o(\omega_1) + n_e(\theta,\omega_1)]$$

won't necessarily be able to solve analytically

Phase Matching (sum frequency generation)

$$P_i^{(2)}(\omega_1+\omega_2) = \epsilon_0 \chi^{(2)}_{ijk}(\omega_1+\omega_2 : \omega_1, \omega_2) \cdot \underline{E_j(\omega_1) E_k(\omega_2)}$$

This is $k(\omega_1) + k(\omega_2)$

generates

$E(\omega_1+\omega_2)$

$\downarrow$

$k(\omega_1+\omega_2)$

so want:

$$k(\omega_1+\omega_2) = k(\omega_1) + k(\omega_2)$$

Possible Phase Matching Configurations

	$E_j(\omega_1)$	$E_k(\omega_2)$	$E_i(\omega_1+\omega_2)$	
Type I	ord	ord	ext	$n_e < n_o$
	ext	ext	ord	$n_o < n_e$
Type II	ord	ext	ext	$n_e < n_o$
	ext	ord	ext	
	ord	ext	ord	$n_o < n_e$
	ext	ord	ord	

Looking more at 2nd order processes

$$P_i^{(2)}(\omega_1+\omega_2) = \epsilon_0 \chi^{(2)}_{ijk}(\omega_1+\omega_2 : \omega_1, \omega_2) E_j(\omega_1) E_k(\omega_2)$$

$E_j(\omega_1) \rightarrow$, $E_k(\omega_2) \rightarrow$ [ ] $\rightarrow P_i(\omega_1+\omega_2)$

since we are putting energy into $P_i(\omega_1+\omega_2)$ there must be a process by which we are loosing energy from E_j & E_k

$E_k(-\omega_2) \rightarrow$, $E_i(\omega_1+\omega_2) \rightarrow$ [ ] $\rightarrow P_j(\omega_1)$ to reduce energy of $E(\omega_1)$

$E_i(\omega_1+\omega_2) \rightarrow$, $E_j(-\omega_1) \rightarrow$ [ ] $\rightarrow P_k(\omega_2)$ to reduce energy of $E(\omega_2)$

There are three processes which must exist (in order to conserve energy)

$\to$ There are 3 equation & 3 nonlinear envelope equations

So somehow these processes must be related.

What is the relationship between processes?

Use anharmonic oscillator model.

$$\chi^{(2)}_{ijk}(\omega_1+\omega_2 : \omega_1, \omega_2) \sim \left[\omega_0^2 - (\omega_1+\omega_2)^2 - i(\omega_1+\omega_2)\Gamma\right]^{-1} \times \left[\omega_0^2 - \omega_1^2 - i\omega_1\Gamma\right]^{-1} \left[\omega_0^2 - \omega_2^2 - i\omega_2\Gamma\right]^{-1}$$

Permute Fields to describe other processes:

i.e. $E_i(\omega_1+\omega_2) \to E_j(\omega_1)$

$E_j(\omega_1) \to E_k(-\omega_2)$

$E_k(\omega_2) \to E_i(\omega_1+\omega_2)$

$$\chi^{(2)}_{jki}(\omega_1 : -\omega_2, \omega_1+\omega_2) \sim \left[\omega_0^2 - \omega_1^2 - i\omega_1\Gamma\right]^{-1} \times \left[\omega_0^2 - \omega_2^2 + \underline{i\omega_2\Gamma}\right]^{-1} \left[\omega_0^2 - (\omega_1+\omega_2)^2 - i(\omega_1+\omega_2)\Gamma\right]^{-1}$$

expression is identical but for damping

If damping can be neglected (no loss)

Then

$$\chi^{(2)}_{ijk}(\omega_1+\omega_2 : \omega_1, \omega_2) = \chi_{jki}(\omega_1 : -\omega_2, \omega_1+\omega_2)$$

and also

$$= \chi^{(2)}_{kij}(\omega_2 : \omega_1+\omega_2, -\omega_1)$$

This is energy conservation symmetry.

3 processes are described by same

χ_{ijk}