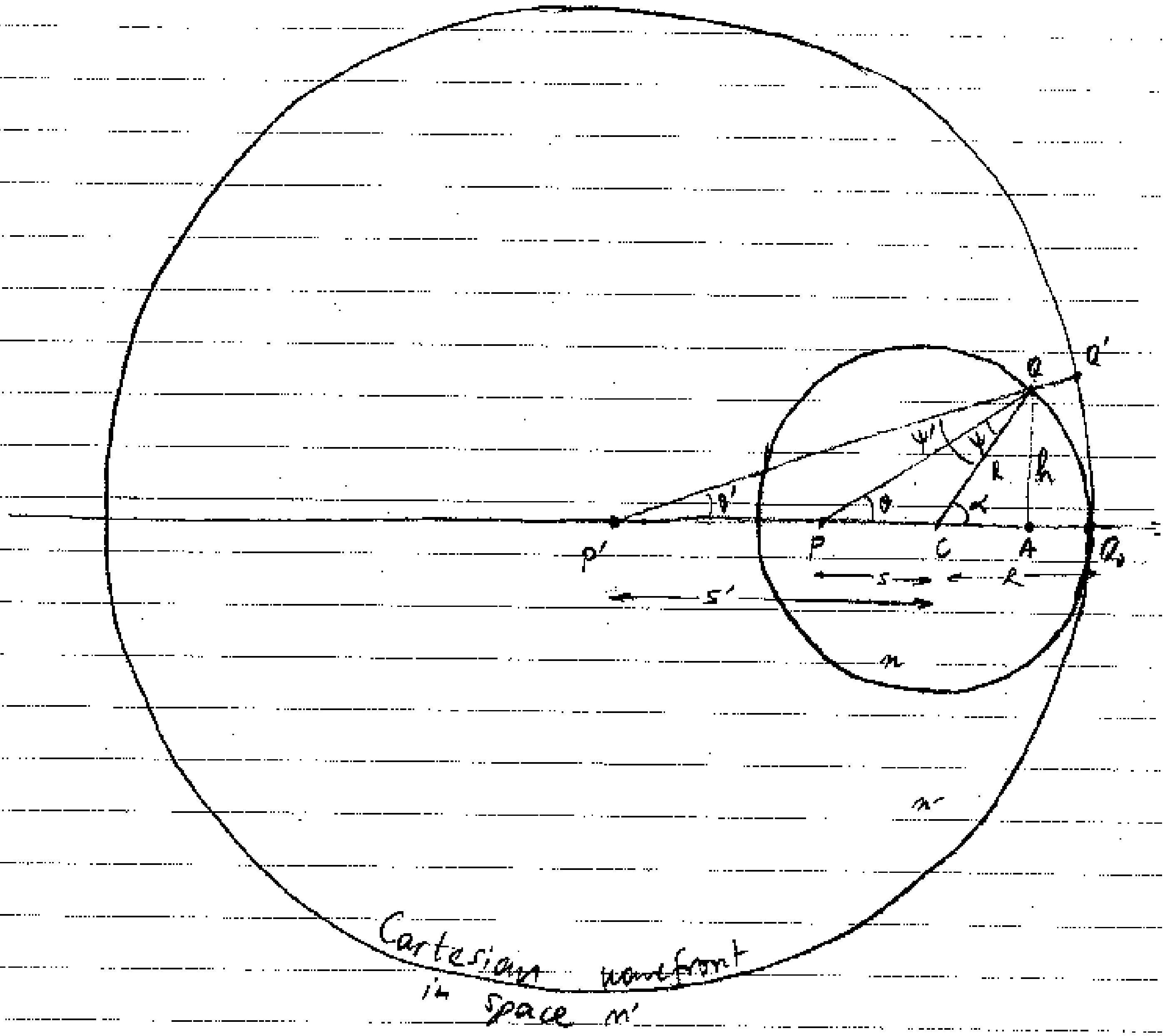


We seek to prove $n^2 S = n'^2 S' = nn'R$.

Note that these are equivalent to: $R = \frac{nS}{n'} = \frac{n'S'}{n}$. (1)

From the on-axis ray we have: $OPL_{n'}(P'Q_0) = OPL_n(PQ_0) \iff$

$\iff n'\overline{P'Q_0} = n\,\overline{PQ_0} \iff n'(s'+R) = n(s+R)$ (2)

From the off-axis ray we have: $OPL_{n'}(P'QQ') = OPL_n(PQQ') \iff$

$\iff n'(\overline{P'Q} + \overline{QQ'}) = n\,\overline{PQ} + n'\overline{QQ'} \iff n'\overline{P'Q} = n\,\overline{PQ}$

KF 3,9 p. 2
solution

$$\Leftrightarrow \frac{n'^2}{n^2} = \frac{\overline{PQ}^2}{\overline{P'Q}^2} \overset{\text{use } \triangle PAQ,\ \triangle P'AQ}{\Leftrightarrow} \frac{n'^2}{n^2} = \frac{\overline{PA}^2 + \overline{AQ}^2}{\overline{P'A}^2 + \overline{AQ}^2}$$

$$\Leftrightarrow \frac{n'^2}{n^2} = \frac{(s + R\cos\alpha)^2 + R^2\sin^2\alpha}{(s' + R\cos\alpha)^2 + R^2\sin^2\alpha} = \frac{s^2 + R^2\cos^2\alpha + 2Rs\cos\alpha + R^2\sin^2\alpha}{s'^2 + R^2\cos^2\alpha + 2Rs'\cos\alpha + R^2\cos^2\alpha}$$

$$= \frac{s^2 + R^2 + 2Rs\cos\alpha}{s'^2 + R^2 + 2Rs'\cos\alpha} \quad (3)$$

It suffices to show that (2) & (3) are satisfied provided (1) holds. Indeed,

$$(2) \overset{(1)}{\Leftrightarrow} n's' + n'\frac{ns}{n'} = ns + n\frac{n's'}{n} \Leftrightarrow n's' + ns = ns + n's' \quad \text{ok}$$

$$(3) \Leftrightarrow n'^2 s'^2 + n'^2 R^2 + 2n'^2 R s'\cos\alpha = n^2 s^2 + n^2 R^2 + 2n^2 R s\cos\alpha \overset{(1)}{\Leftrightarrow}$$

$$\Leftrightarrow n^2 ss' + n'^2\frac{n^2 s^2}{n'^2} + 2n'^2\frac{ns}{n'}s\cos\alpha = n'^2 ss' + n^2\frac{n'^2 s'^2}{n^2} + 2n^2\frac{n's'}{n}s\cos\alpha$$

$$\Leftrightarrow n^2 ss' + n^2 s^2 + 2n'n s's\cos\alpha = n'^2 ss' + n'^2 s'^2 + 2nn'ss'\cos\alpha$$

$$\Leftrightarrow n^2 s(s' + s) = n'^2 s'(s + s') \overset{(1)}{\Leftrightarrow} s' + s = s + s' \quad \text{ok}.$$

We proved that (1) leads to (OPL) equality for on- and off-axis rays (independent of α). $\square$

better solution

$$\theta + (\pi - \alpha) + \gamma = \pi \Rightarrow \gamma = \alpha - \theta$$

$$\gamma' = \alpha - \theta'$$

$$n \sin\gamma = n' \sin\gamma' \Rightarrow n \sin(\alpha - \theta) = n' \sin(\alpha - \theta')$$

$$\rightarrow n\left(\sin\alpha\cos\theta - \cos\alpha\sin\theta\right) = n'\left(\sin\alpha\cos\theta' - \cos\alpha\sin\theta'\right)$$

$$\Rightarrow n\left(\frac{h}{R}\cdot\frac{s+\cancel{R}}{\overline{PQ}} - \cancel{\frac{\ell}{R}\cdot\frac{h}{\overline{PQ}}}\right) = n'\left(\frac{h}{R}\cdot\frac{s'+\cancel{R}}{\overline{PQ}} - \cancel{\frac{\ell}{R}\frac{\ell}{\overline{PQ}}}\right)$$

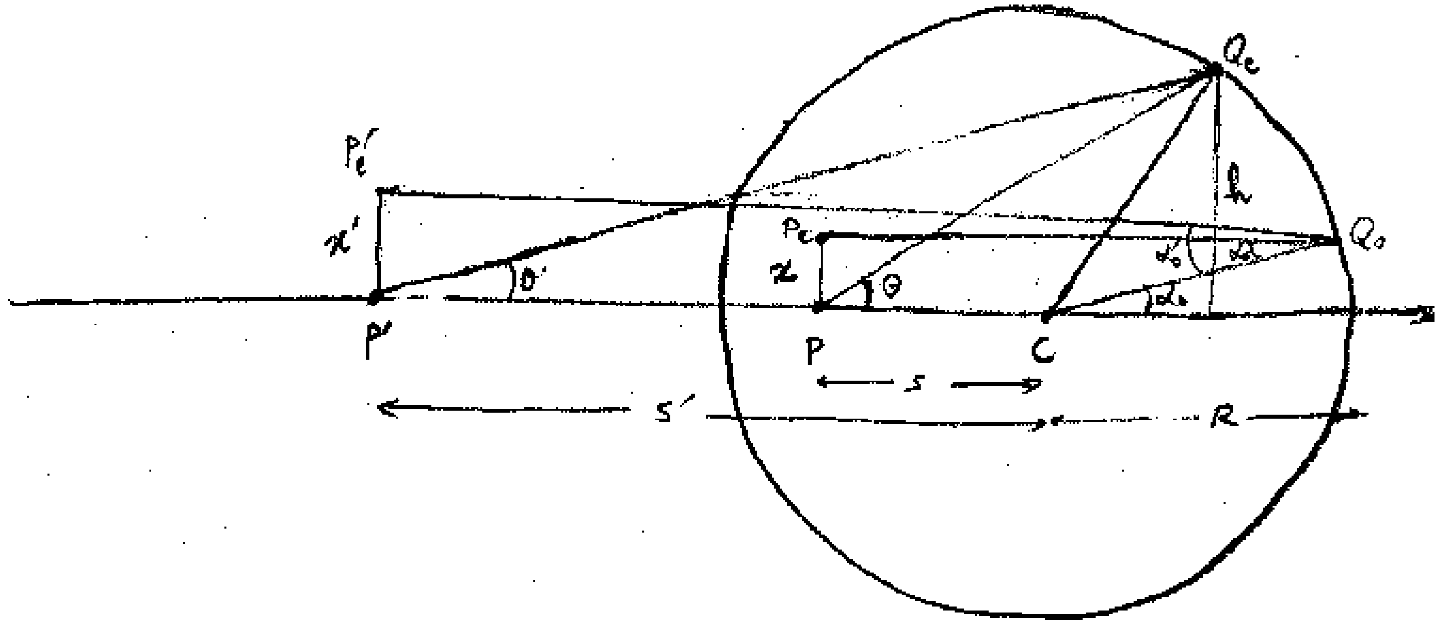
$$\left.\begin{array}{l} \Rightarrow \dfrac{ns}{\overline{PQ}} = \dfrac{ns'}{\overline{P'Q}} \\ \text{imaging condition: } \quad n\overline{PQ} = n'\overline{P'Q} \end{array}\right\} \Rightarrow n^2 s = n'^2 s'$$

on-axis: $\quad n'(s'+R) = n(s+R) \Rightarrow n'\cdot\dfrac{n^2 s}{n'^2} + n'R = ns + nR$

$$\Rightarrow \left(\frac{n^2}{n'} - n\right) s = (n - n')R \Rightarrow n\,\frac{n - n'}{n'}\,s = (n - n')R \Rightarrow$$

$$\Rightarrow R = \frac{n}{n'} s \qquad \square$$

KF 4.33
solution



$$n x \sin\theta = n x \frac{h}{\overline{PQ_e}} = \frac{n^2 x h}{(n\overline{PQ_e})}$$

$$n' x' \sin\theta' = n' x' \frac{h}{\overline{P'Q_e}} = \frac{n'^2 x' h}{(n'\overline{P'Q_e})}$$

suff. to show that $n^2 x = n'^2 x'$

Consider the horizontal ray from $P_e \to Q_0$ ← easier to do the math!

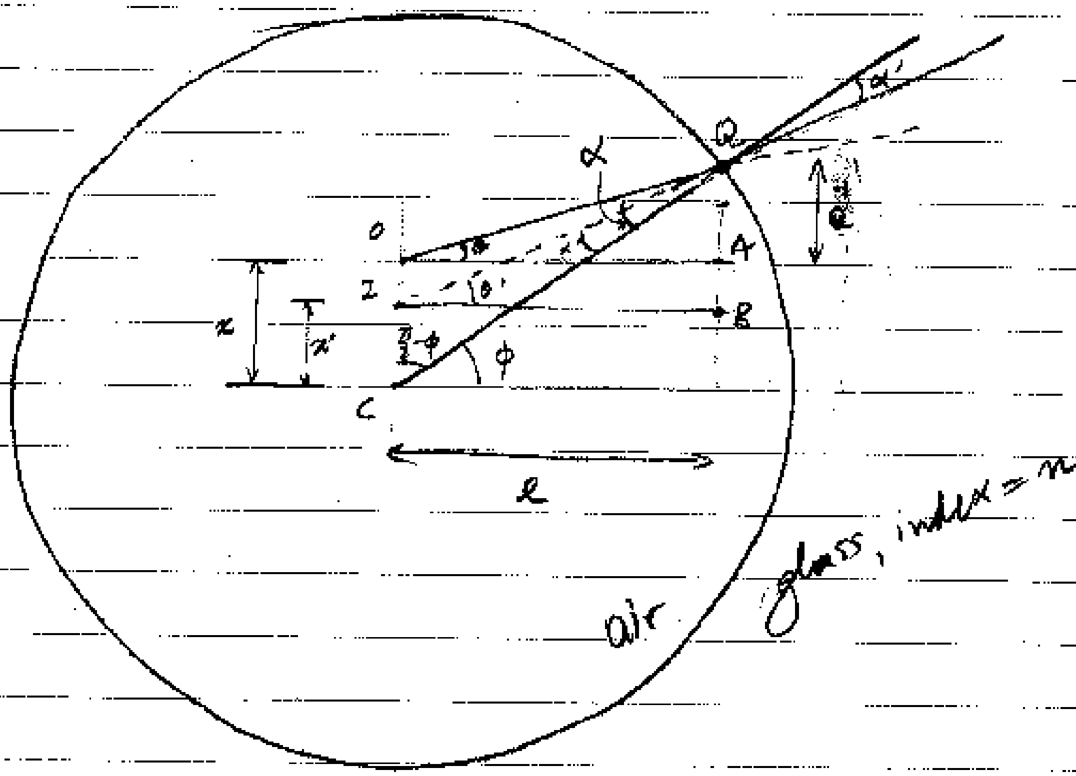
Snell's law: $n \sin\alpha_0 = n' \sin\alpha_0' \Rightarrow n\alpha_0 = n'\alpha_0'$ [permitted sin
$\tan\alpha_0 = \frac{x}{R} + O(x^2$

Now we have: $x = R\alpha_0$

$$x' = x + (R + s')(\alpha_0' - \alpha_0) \xlongequal{\text{use } s' = \frac{Rn}{n'}} R\alpha_0 + \left(R + \frac{n}{n'}R\right)\left(\frac{n}{n'}\alpha_0 - \alpha_0\right)$$

$$= R\alpha_0\left(1 + \frac{n}{n'} - 1 + \frac{n^2}{n'^2} - \frac{n}{n'}\right) = \frac{n^2}{n'^2} R\alpha_0 = \frac{n^2}{n'^2} x \qquad \square$$

KF 4.34
solution



sine law on $\triangle COQ$: $\frac{x}{\sin\alpha} = \frac{\overline{OQ}}{\sin(\frac{\pi}{2}-\phi)} \Rightarrow \cos\phi = \overline{OQ}\,\frac{\sin\alpha}{x}$

sine law on $\triangle CIQ$: $\frac{x'}{\sin\alpha'} = \frac{\overline{IQ}}{\sin(\frac{\pi}{2}-\phi)} \Rightarrow \cos\phi = \overline{IQ}\,\frac{\sin\alpha'}{x'}$

Snell's law: $n\sin\alpha' = \sin\alpha$

$\Rightarrow \overline{OQ}\,\frac{n\sin\alpha'}{x} = \overline{IQ}\,\frac{\sin\alpha'}{x'} \Rightarrow nx'\,\overline{OQ} = x\,\overline{IQ}$

orth. $\triangle OAQ$: $\overline{OQ} = \frac{a}{\sin\theta}$

orth: $\triangle IBQ$: $\overline{IQ} = \frac{a+(x'-x)}{\sin\theta'}$

$\Rightarrow \frac{nx'a}{\sin\theta} = \frac{xa}{\sin\theta'} + \underbrace{O(x^2)}_{\text{DROP}} \Rightarrow nx'\sin\theta' = x\sin\theta \Rightarrow O, I$ conjugate

$n>1 \Rightarrow |x|>|x'|$.