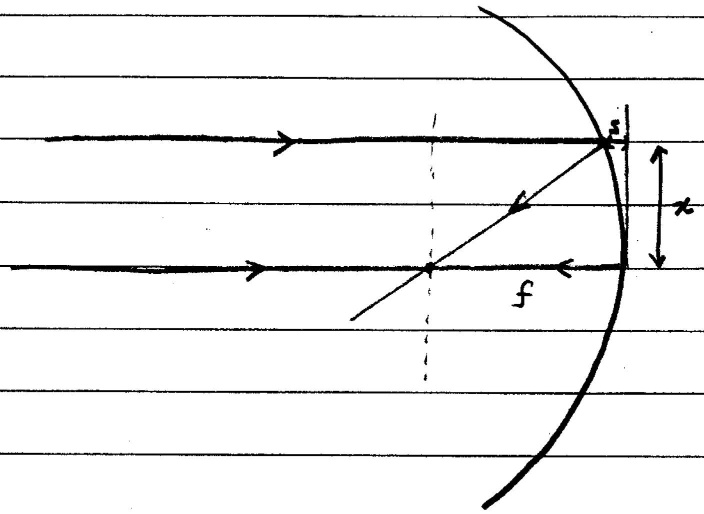
QUIZ solutions.

1.

1.a Parabolic mirror

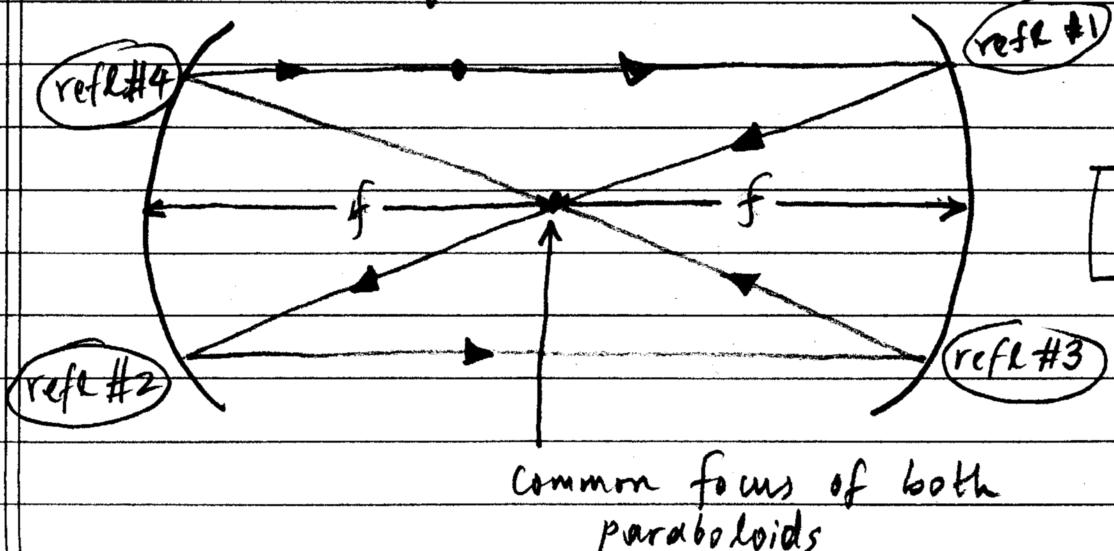


$$(f-s)+\sqrt{(f-s)^2+x^2} = 2f \Rightarrow (f-s)^2+x^2=(f+s)^2$$

$$\Rightarrow x^2 = (f+\cancel{s}+f-\cancel{s})\cdot(\cancel{f}+s-\cancel{f}+s)$$
$$= 4fs \Rightarrow$$

$$\Rightarrow s = \frac{x^2}{4f}$$ ← focal length [location of focus]

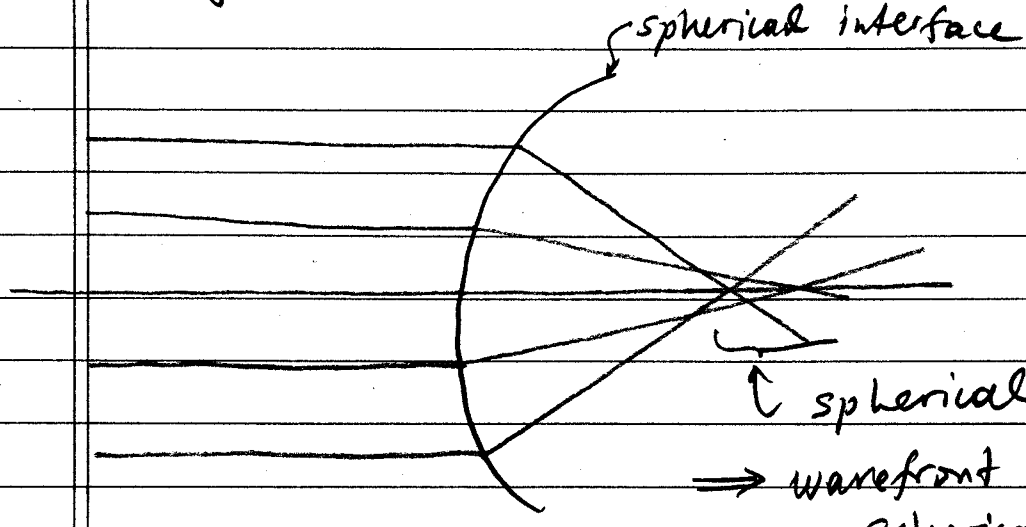
Self-retracing condition



$\boxed{\therefore\ d=2f}$

(2) (2.a) Not correct

E.g. Normal incidence



spherical aberration

$\Rightarrow$ wavefront is not quite spherical!

(in fact it is spherical + ρ^4 quartic aberration)

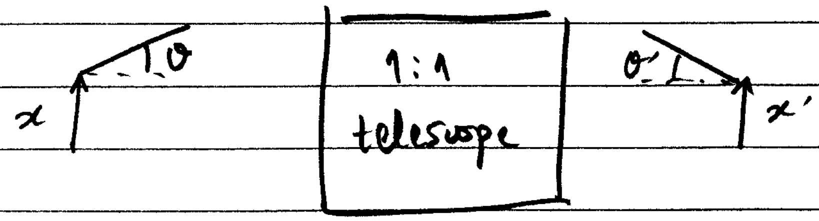
Oblique incidence is even worse (coma is introduced as well).

(2.b) True!

In a telescope $m_\theta = \dfrac{1}{m_x} = 1$ (since 1:1 and immersed in free space)

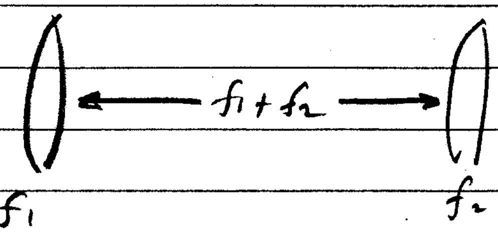
$m_\theta = 1 \Rightarrow \theta = \theta' \Rightarrow \sin\theta = \sin\theta'$

$m_x = 1 \Rightarrow x = x'$

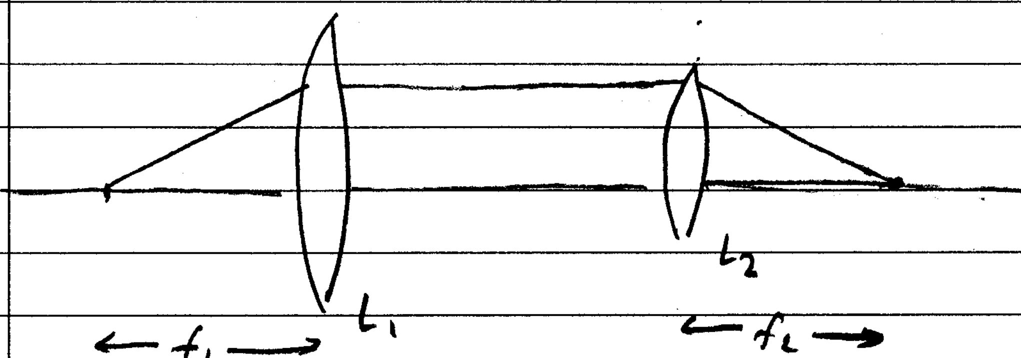


$x\sin\theta = x'\sin\theta'$
Abbe's condition is satisfied
$\Rightarrow$ aplanatic.

③ Observe that the optical system is a <u>telescope</u>

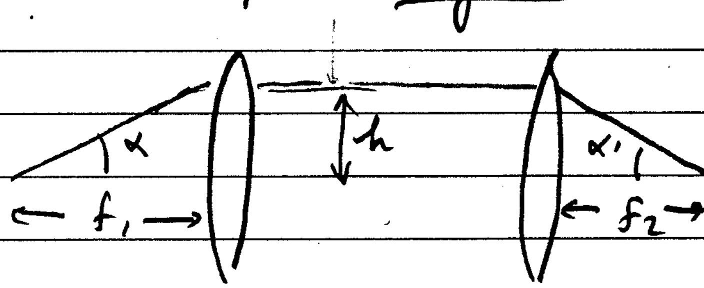


(3.a) Ray-trace through



$\therefore$ image is located 5 cm behind L_2

(3.b) First compute <u>angular</u> magnification

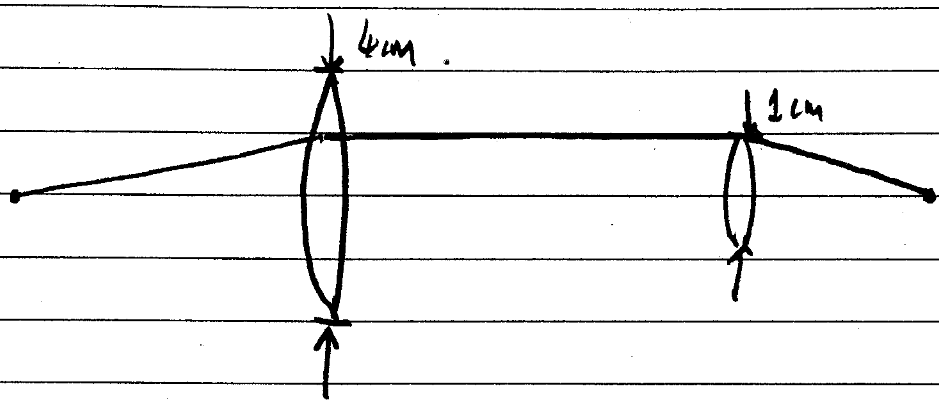


$h_1 = f_1 \alpha$ (triangle left)
$h = f_2 \alpha'$ (triangle right)

$$\Rightarrow m_\alpha = \frac{\alpha'}{\alpha} = \frac{f_1}{f_2} = 2$$

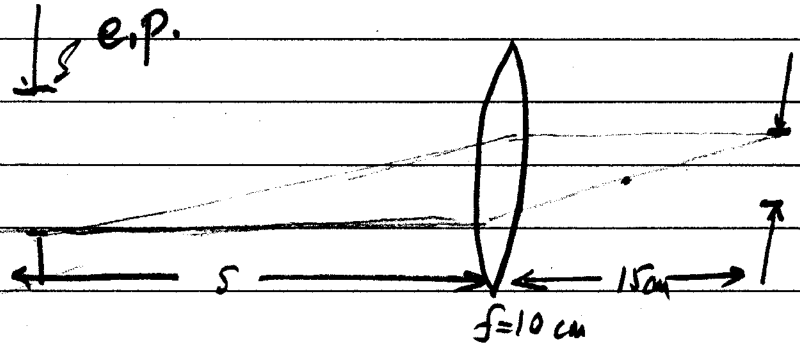
<u>lateral</u> magnification $m_x = \dfrac{1}{m_\alpha} = 0.5$

3.c marginal ray: (**Note**: not to scale!)



limited by 2nd lens $\Rightarrow$ rim of L_2 is the **aperture** stop

image aperture through L_1

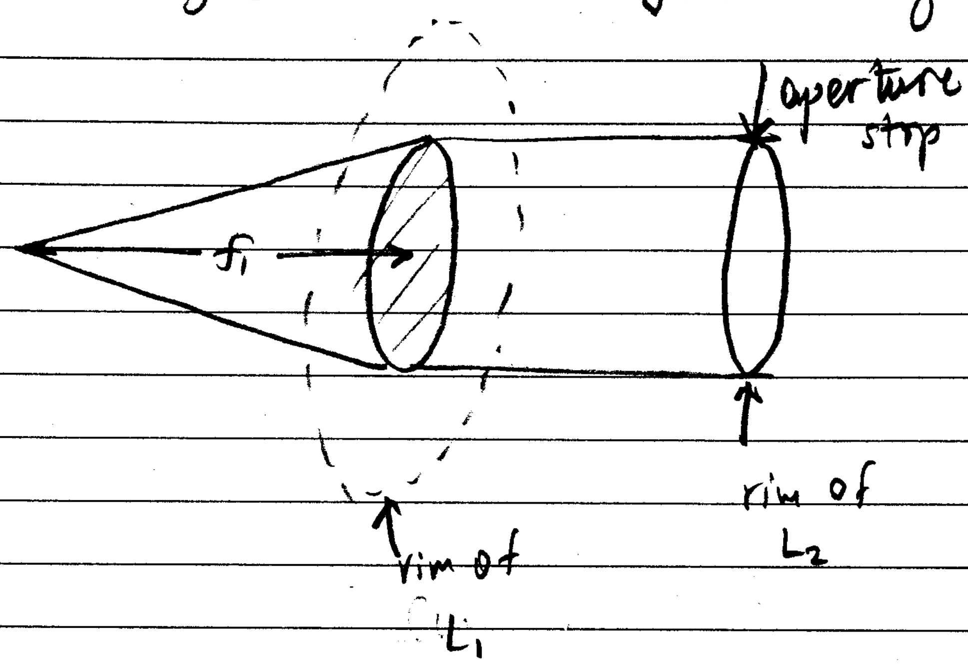


$$\frac{1}{s}+\frac{1}{15}=\frac{1}{10} \Rightarrow \frac{1}{s}=\frac{15-10}{150}=\frac{1}{30} \Rightarrow s=30\,\text{cm}$$

$$\text{size of e.p.} = \underbrace{\frac{30\,\text{cm}}{15\,\text{cm}}}_{\frac{s}{s_{in}}} \times \underbrace{1\,\text{cm}}_{\text{aperture size}} = 2\,\text{cm} \text{ (diameter)}$$

(3.d) The source emits power W into solid angle 2π (= half-space)

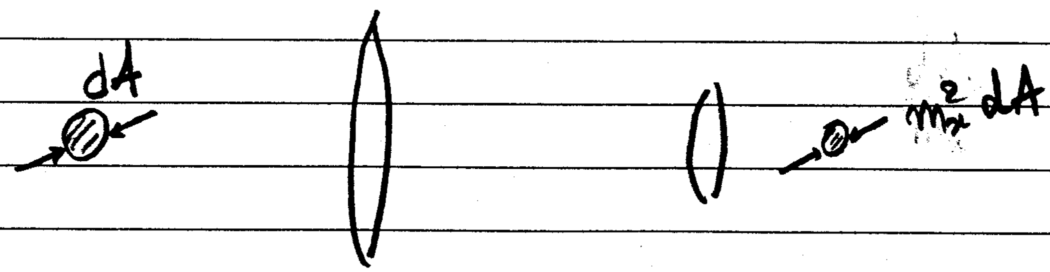
solid angle subtended by the imaging system:



$$\Omega = \frac{\text{area of } L_1}{f_1^2} = \frac{\pi\,(.5\text{cm})^2}{(10\text{cm})^2} = .0025\,\pi$$

power exiting the system: $W \cdot \dfrac{\Omega}{2\pi} = 1.25 \times 10^{-3}\, W$

(3.e)



size of imaged source: $(dA)\cdot m_x^2 = (0.5)^2 \cdot (dA)$

$$\left(\begin{array}{c}\text{irradiance}\\ \text{@ image}\end{array}\right) = \frac{\left(\begin{array}{c}\text{power}\\ \text{@ image}\end{array}\right)}{\left(\begin{array}{c}\text{image}\\ \text{area}\end{array}\right)} = \frac{1.25 \times 10^{-3}\, W}{(0.5)^2 (dA)} = 5 \times 10^{-3} \frac{W}{(dA)}$$

$$= 5 \times 10^{-3} \left(\begin{array}{c}\text{radiant exitance}\\ \text{at source}\end{array}\right)$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.717J/MAS.857J Optical Engineering **Fall '00**

QUIZ Thursday, Oct. 19, 2000

Duration: 50 min (=1 academic hour)

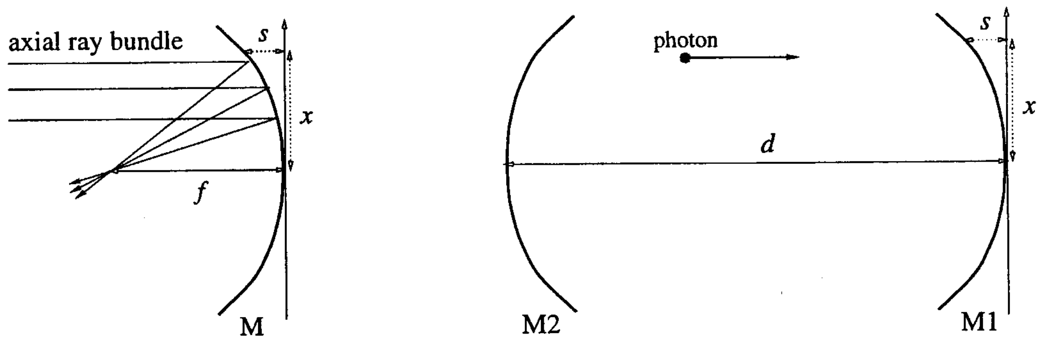
Instructions: All aids are allowed except the use of communication of any kind. Use of results derived in the textbook, lectures, recitations, and homeworks is encouraged. Among all possible correct solutions, always seek the simplest one.

1. (30%) Paraboloidal reflectors.

1.a) Consider a parallel ray bundle propagating along the optical axis and being reflected by a mirror M, as shown in the left-hand figure below. Show that the bundle comes to a perfect focus at distance f from the center of M, provided that the shape of M is given (in the cross-section shown) by

$$s(x) = \frac{x^2}{4f}. \tag{1}$$

[Hint: establish equality of appropriate optical paths of the axial ray and an arbitrary ray displaced by x with respect to the optical axis.]



1.b) Mirror M1 has shape $s(x)$ as in (1). Mirror M2 is identical to M1 and the two are aligned so that they face each other and share a common optical axis, as shown in the right-hand figure above. A photon finds itself between the two mirrors, propagating parallel to the common axis of the two mirrors. Determine the distance d between the centers of the two mirrors such that the photon retraces its path after exactly four reflections from the mirror surfaces. Trace the photon path (*i.e.*, the optical ray) according to your solution.

2. (20%) Comment on the validity of the following statements.

   2.a) A plane wave refracted at the spherical interface between two dielectric media becomes a spherical wave (convergent or divergent.)

   2.b) A telescope in free space with 1:1 magnification is aplanatic.

3. (50%) An infinitessimally small planar Lambertian light source has area $d\mathcal{A}$ and radiates power W [Watts] in the right half-space. Part of the radiation is captured by the imaging system composed of thin lenses L1, L2 as shown in the figure below, and forms a real image of the source to the right of lens L2. The blocks surrounding the lenses extend laterally all the way to infinity. Use paraxial geometric optics to answer the following questions:

   3.a) Where is the real image located?

   3.b) What is the lateral magnification of the image?

   3.c) Where are the aperture stop and the entrance pupil?

   3.d) What is the radiant flux received at the image plane?

   3.e) What is the ratio of irradiance at the image location to the radiant exitance of the source?