6.824 Artificial Intelligence, Fall 1992
Problem Set 3

Due: Beginning of class Wednesday, October 7

Problem 1. (25 points) This problem is on Hidden Markov Models. Consider a hidden Markov model λ with four internal state a_1, a_2, a_3 and b and two external symbols a and b. We assume that if the internal state is either a_1, a_2, or a_3 then the external symbol is always a and if the internal state is b then the external symbol is always b. We also assume that the state transition probability matrix is determined by four numbers A_1, A_2, A_3 and B. A_1 is the probability that when the machine is in state a_1 then the next machine state remains a_1. A_2, A_3 and B are the analogous "no change" probabilities for states a_2, a_3 and b respectively. If the machine does change state it must change from an "a" state to a "b" state or vice versa, e.g., there is no chance of a transition from a_1 to a_2. When the machine changes from state b to one of the a states all three a states are equally likely. Initially all four states are equally likely.

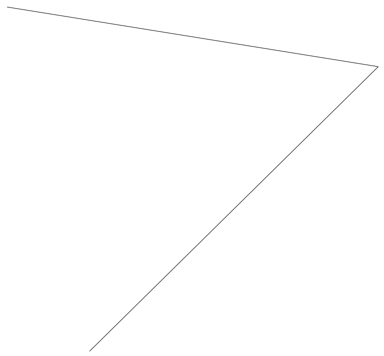
Part a. Let D be a sequence of k_1 a's then k_2 b's then k_3 a's then k_4 b's then k_5 a's and finally k_6 b's. Define a sequence S of $k_1+k_2+k_3+k_4+k_5+k_6$ internal states and values for A_1, A_2, A_3, and B such that $P_r(S|\lambda \wedge D)$ is maximized (λ is the hidden Markov model determined by A_1, A_2, B_1 and B_2).

Part b. State transitions between the states a_1, a_2 and a_3 are not allowed in the above model. Is this property stable under Baum-Welch reestimation? In other words, if we use Baum-Welch reestimation to construct a new model, will this new model allw transitions between different a states? Explain your answer.

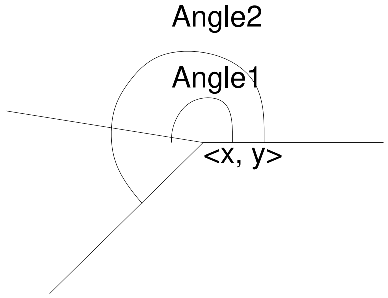
Part c. The model you constructed in part a generates the external symbols deterministically — only one external symbol is possible for a given internal state. Is this property preserved by Baum-Welch reestimation?

Part d. Describe a sequence of external symbols such that Baum-Welch reestimation starting from the above model would move to a model where the transition from the b to an a state does not give the a states equal probability. Explain your answer.

Problem 2. This problem is on the use of MAP in vision. It would be a good idea to read this problem before reading Well's paper since the example given here might make the general formalism in the paper clearer. Suppose that we have "features" that are corners, i.e., places where two edges meet such as that shown below.



A corner can be represented by four numbers $<x,\ y,\ \Theta,\ \Psi>$ as shown below (in the figure Θ and Ψ are called angle1 and angle2).



Suppose that we are looking for a square in some image. A square has four corners so we have a model vector

$$M \ = \ \{M_1,\ M_2,\ M_3,\ M_4\}$$

where each M_i is a model corner $<x_{M_i},\ y_{M_i},\ \Theta_{M_i},\ \Psi_{M_i}>$. We will take a pose of the triangle to be a translation plus a rotation. A pose can therefore be specified by three numbers $<x,\ y,\ \Delta>$ where Δ is a (counter clockwise) rotation angle.

part a. Give an expression for the angle $P(M_i,\ \beta)$ where M_i is a model angle and β is a pose as specified above. $P(M_i,\ \beta)$ consists of four numbers that specify how the model angle M_i appears under the pose β.

Now assume that a set of angles is generated by running a feature detection algorithm (an angle detector) on an image. Let $<Y_1,\ \ldots,\ Y_n>$ be the set of angles detected in a given image. Each Y_i consists of four numbers — the four numbers that specify an angle. Let Γ be the tuple $<\Gamma_1,\ \ldots,\ \Gamma_n>$ where Γ_i is an interpretation of angle Y_i. Each Γ_i is one of the four model angles M_j or the special "unassigned" symbol $\perp$. We assume that if Γ_i is M_j then under pose β we have that Y_i is $P(M_j,\ \beta)$, as defined in part a, plus Gaussian noise. Assume that the noise in each of the four numbers specifying Y_i incorporates noise independent of the noise in the other components and the noise in each component has zero mean and variance σ_p for noise in positions and σ_a for noise in angles.

Part b. Give an expression for

$$P_r(Y_i|\Gamma,\ \beta)$$

as a function of the three values specifying β, the four values in the model angle Γ_i, and the noise variances σ_p and σ_a. Your answer should be consistent with the equations in Well's paper.