

# Design Considerations for Force Controllable Series Elastic Actuators

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Thesis Committee Meeting

Dec. 7, 1999

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# Agenda

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- What is my goal?
- Thesis Outline
- Past Material
  - EM Series Elastic Actuator (FrAP)
- Hydro-elastic Actuator (HyEAP)
- Non-Linear Springs
- Scaling
- Go Downstairs

# What is my Goal?

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- Several Discussions with people who ask:
  - “How much is bandwidth reduced?”
  - “What spring do I use for my application?”
- Goals
  - Practical Implementation
    - Prototype Actuators
    - Scaling
  - Understand and explain spring choice
    - Stability
    - Large Force Bandwidth
    - Impedance
    - Scaling
    - Non-linear

# Written Thesis Outline

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- Introduction
- Related Work
- Linear Series Elastic Actuators
- Non-Linear Series Elastic Actuators
- Compliant Element
- Actuator Scaling
- Application
- Conclusion

# EM Series Elastic Actuator

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- Mass-Spring-Damper Model
- PD Control
- Investigated
  - Large Force Bandwidth & Motor Saturation
  - Non-Linear Friction Reduction
  - Output Impedance
- Guides for Component and Spring Selection

# Spring Selection

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- Competing desires.
  - Large force bandwidth requires large spring.
  - Non-linear friction and impedance requires small spring.
- Choose minimum operational bandwidth (5-10 Hz).
  - Defined by natural mass spring resonance  $\omega_n$ .
  - Upper bound for  $k_s$ .
- Choose minimum friction and impedance levels.
  - Lower bound for  $k_s$ .

# Hydroelastic Actuator

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- Force controllable hydraulic actuator
- High power density
- Typically very stiff
- Related Work
  - Klinedon & Hogan (1983) Variable pressure pneumatic cylinder at the end of hydraulic cylinder for “tunable impedance”.
  - Wells & Jacobsen (1990) passive accumulators in piston.
  - Alleyne (1997) Lyapunov controller for force control.

# Hydroelastic Goals

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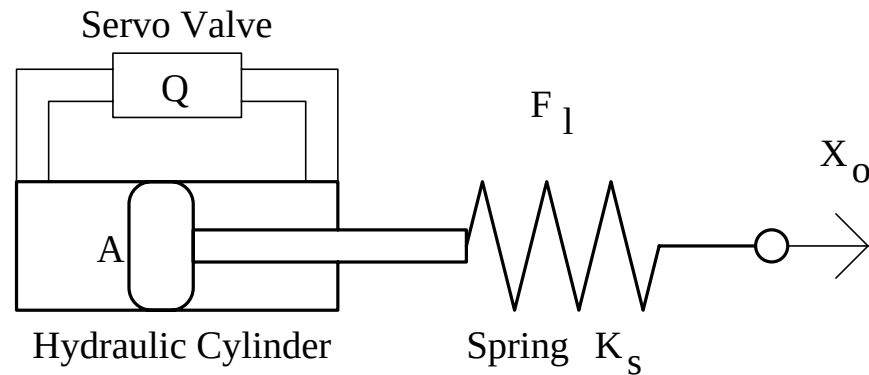
- Simple controller achieves moderate bandwidth.
- Stability with Shock Tolerance.
- Actively Back-drivable.
- Low impedance





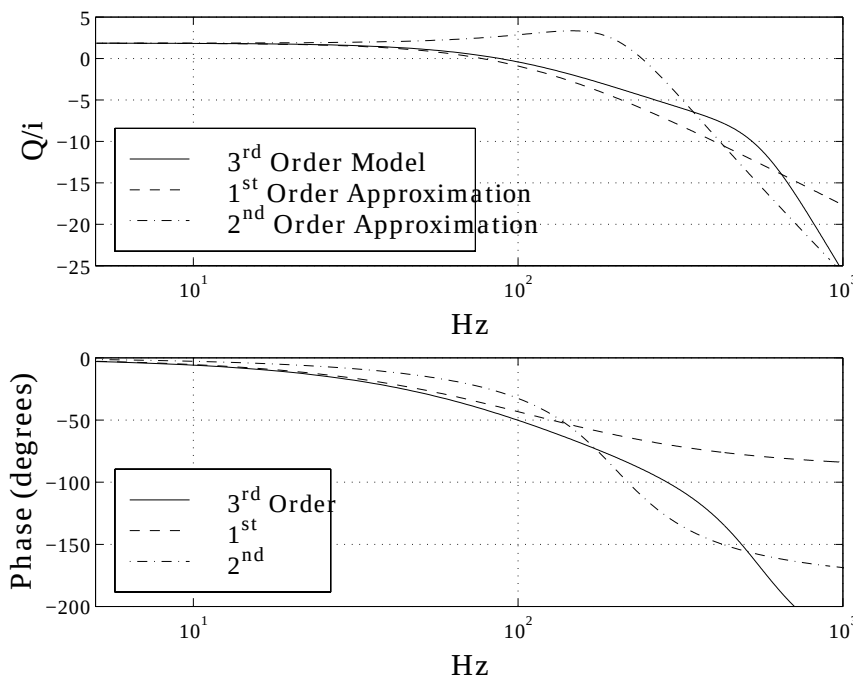
# Actuator Model

- Inputs
  - Fluid Flow
  - Position input
- Assumptions
  - Equal Area
  - Current equals flow rate
  - 1<sup>st</sup> order servo valve
  - No saturation (for now)
  - Incompressible



# Servo Valve

- Typically 3<sup>rd</sup> Order Model
- Used 1<sup>st</sup> order for hand calculations
- Linear Model
- Non-linearity
  - Null bias
  - Threshold

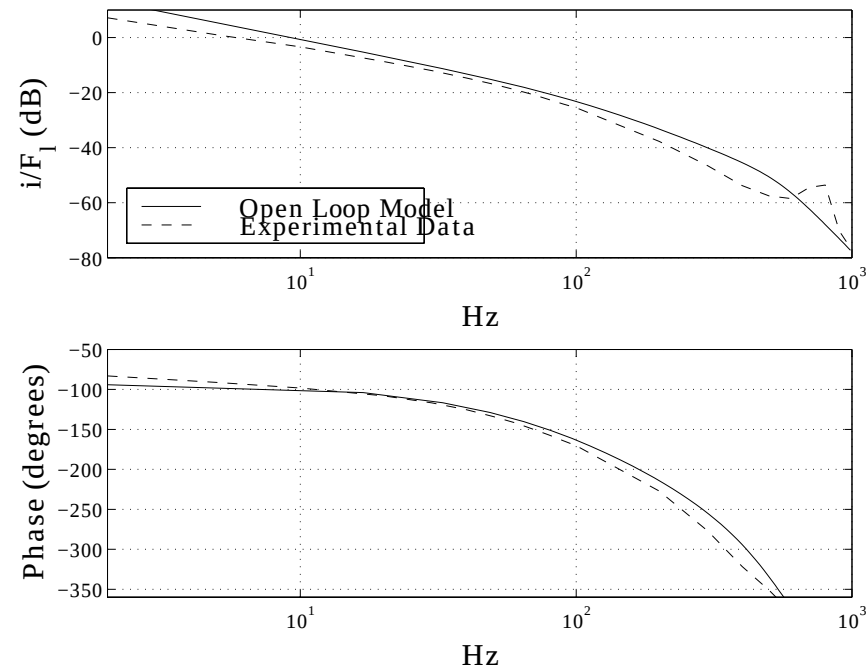
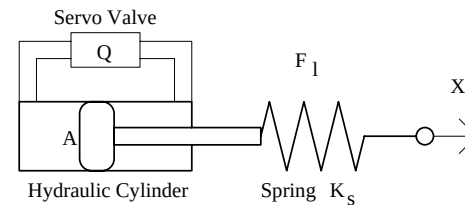


# Servo-Spring OL model

- Flow is hard to measure.
- Spring deflection is easy.
- Fixed Endpoint

$$F_L = k_s \left( \frac{\dot{Q}(i)}{As} - x_0 \right)$$

$$Q(i) = \frac{K_v}{\tau s + 1} i$$



# P Closed Loop Equations

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$$F_l = \frac{F_d - \frac{A(\tau s + 1)}{k_s K K_v} s k_s x_o}{\frac{A\tau}{k_s K K_v} s^2 + \frac{A}{k_s K K_v} s + 1}$$

Closed Loop Equations

$$S = \tau s$$

$$\Omega_p = \sqrt{\frac{k_s K K_v \tau}{A}}$$

Dimensional Groups

$$\frac{F_l}{F_d} = \frac{1}{\frac{S^2}{\Omega_p^2} + \frac{S}{\Omega_p} + 1}$$

Case 1: Fixed End Condition

$$\frac{F_l}{k_s x_o} = \frac{\frac{(S+1)}{\Omega_p^2} S}{\frac{S^2}{\Omega_p^2} + \frac{S}{\Omega_p} + 1}$$

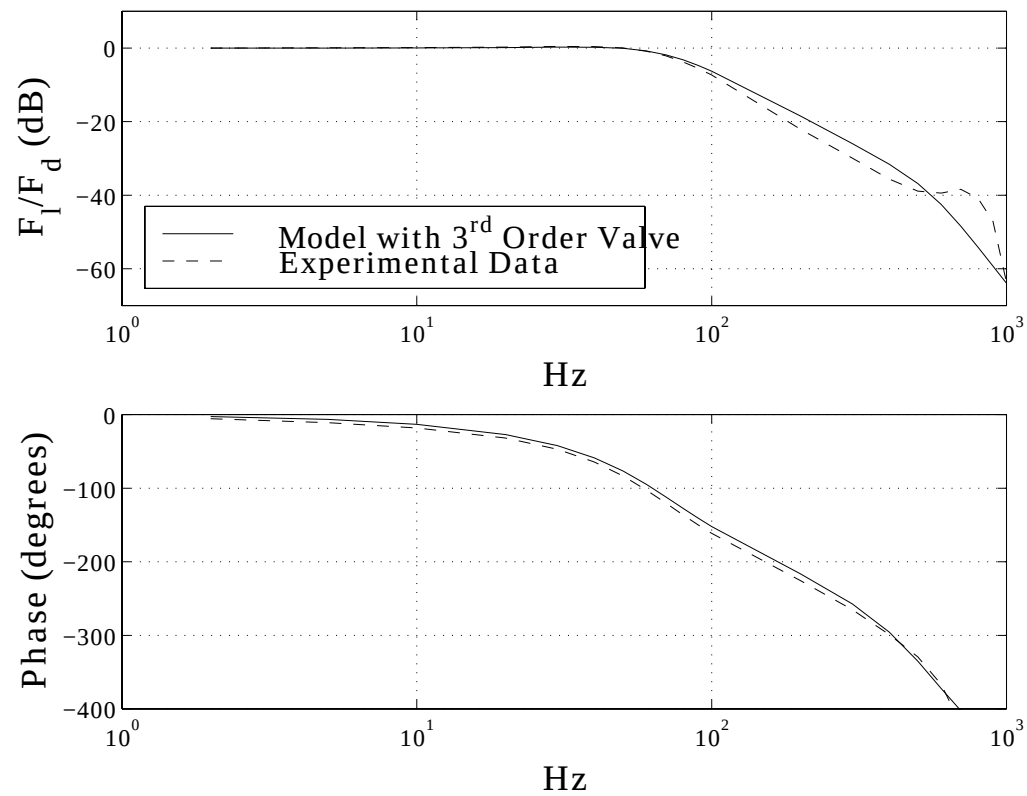
Case 2: Free End Condition

# P Controller

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- Proportional (P)
  - Null bias offset
  - Steady state error despite free integrator in the servo-valve.
  - Unpredictable and moves during operation
- Great Bandwidth
  - Bandwidth and damping are coupled
  - Pick one but not the other

# P Bode Plot



# PI Controller

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- Proportional Integral (PI)
  - Zero steady state error
  - Lighter touch
- Problems
  - Servovalve Threshold problems
  - Valve Stiction and controller windup
- Solved with dither
  - Adds light vibration during free motion

# PI Closed Loop Equations

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$$F_l = \frac{\left( \frac{K_p}{K_i} s + 1 \right) F_d - \frac{A(\tau s + 1)}{k_s K K_i K_v} s^2 k_s x_o}{\frac{A(\tau s + 1)}{k_s K K_i K_v} s^2 + \frac{K_p}{K_i} s + 1}$$

Closed Loop Equations

$$Z = \tau \frac{K_p}{K_i}$$

$$\Omega_{pi} = \tau \sqrt{\frac{k_s K K_i K_v}{A}}$$

Dimensional Groups

$$\frac{F_l}{F_d} = \frac{\left( \frac{1}{Z} S + 1 \right)}{\frac{(S + 1)S^2}{\Omega_{pi}^2} + \frac{S}{Z} + 1}$$

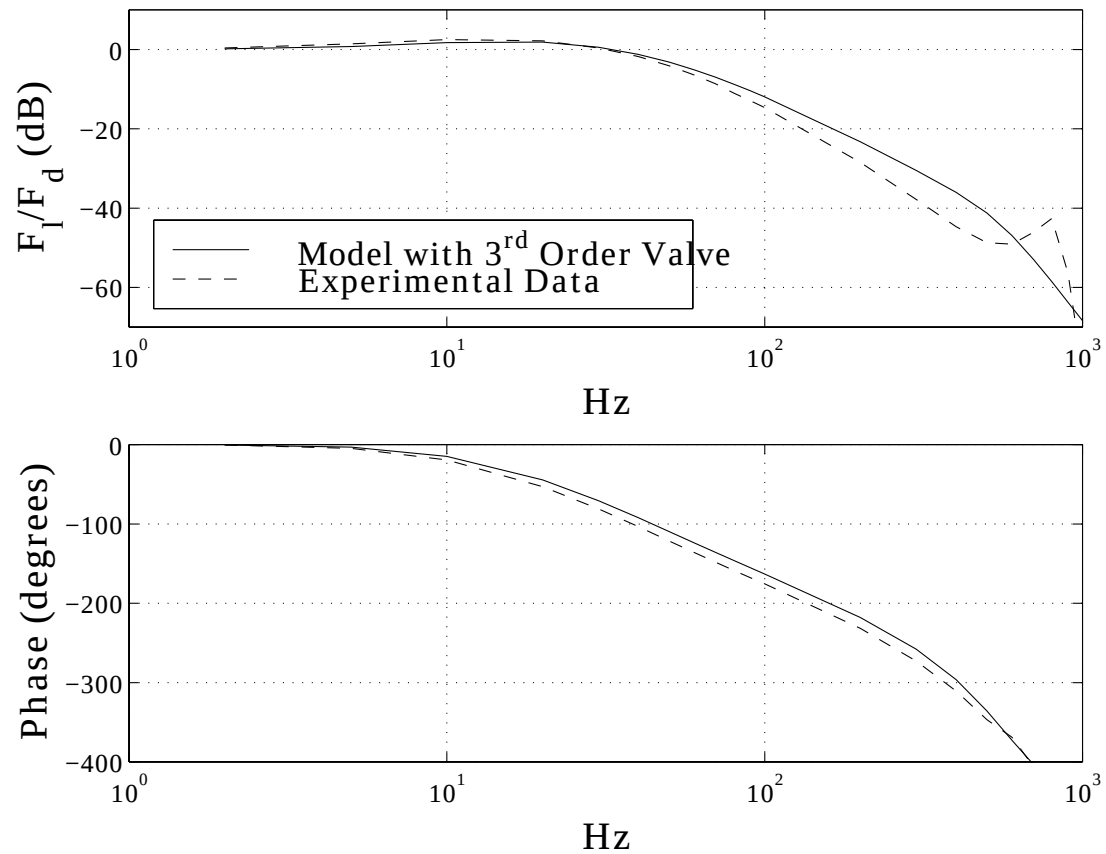
Case 1: Fixed End Condition

$$\frac{F_l}{k_s x_o} = \frac{-\frac{(S + 1)}{\Omega_{pi}^2} S^2}{\frac{(S + 1)S^2}{\Omega_{pi}^2} + \frac{S}{Z} + 1}$$

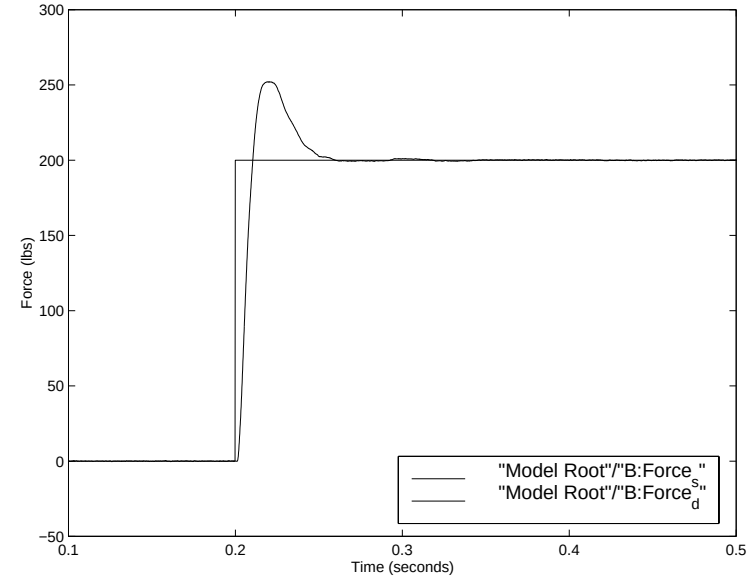
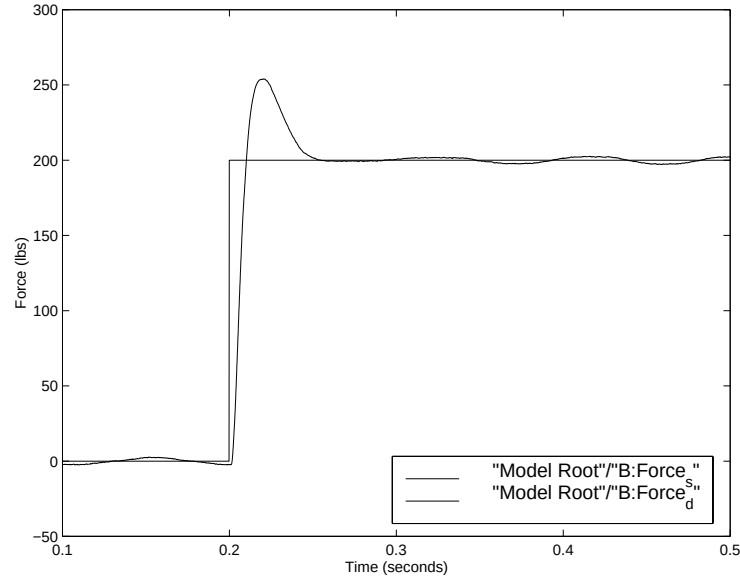
Case 2: Free End Condition



# PI Bode Plot



# Threshold and Dither



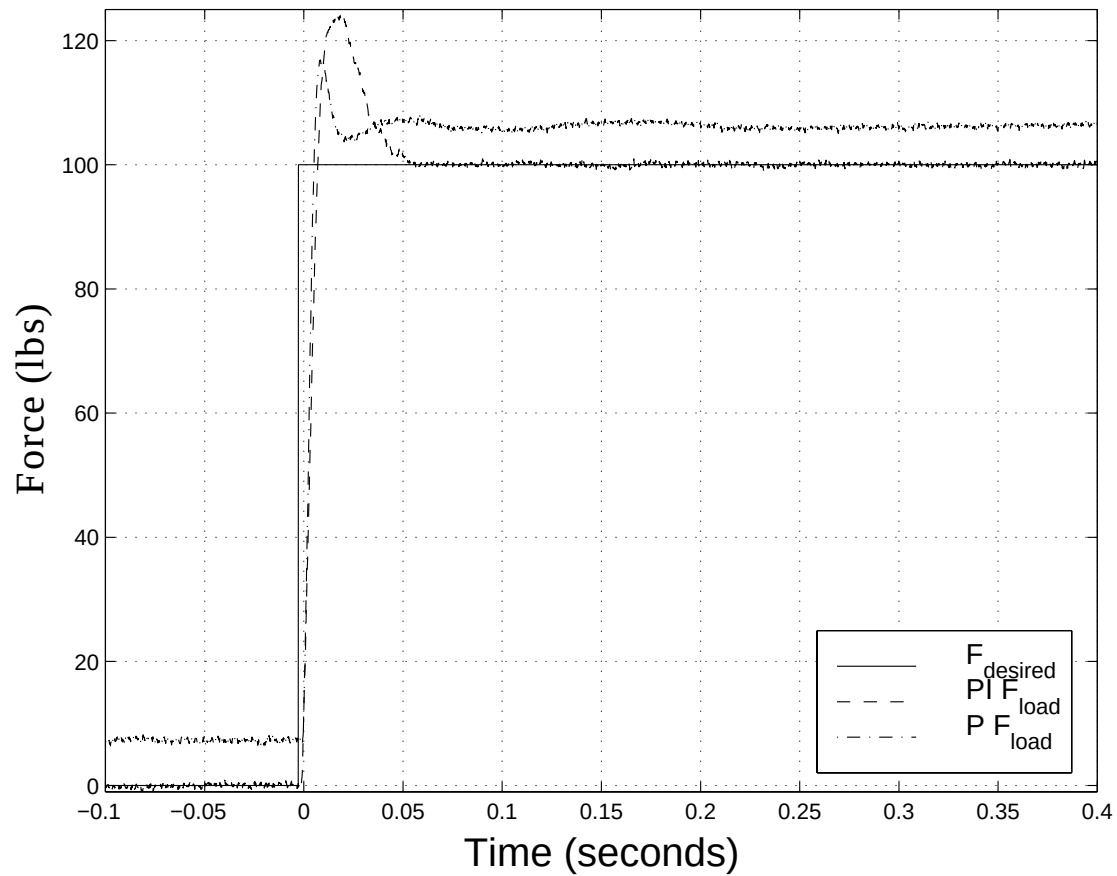
# Controller Compromise

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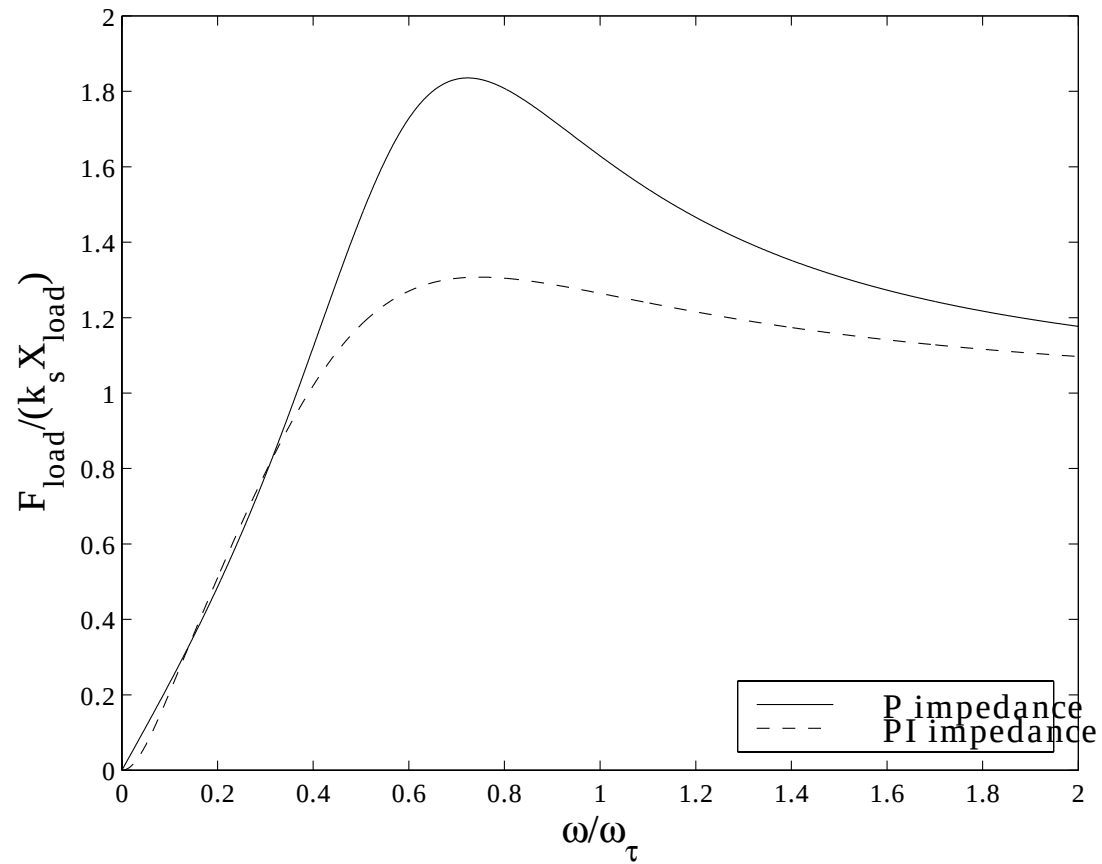
- Integral roll-off

$$\dot{Q} = K \left( K_p + \frac{K_i}{\tau_i s + 1} \right)$$

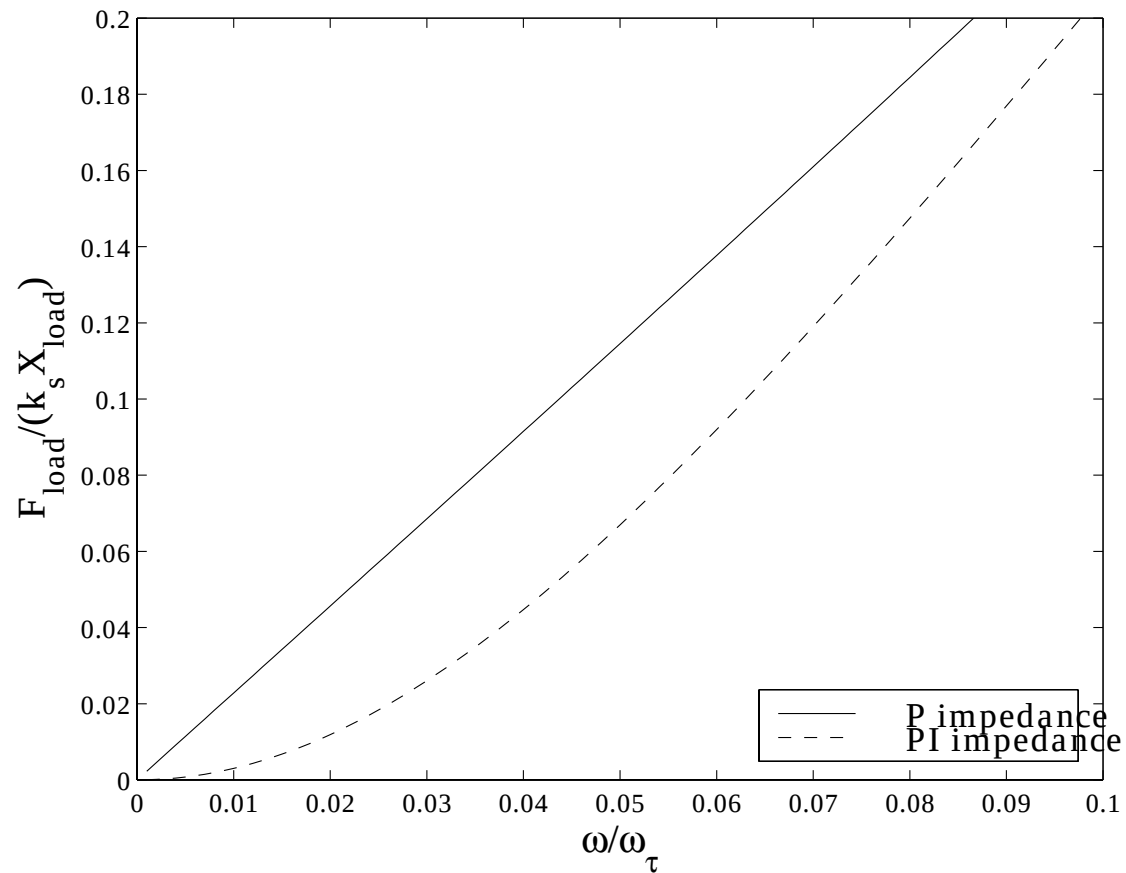
# P and PI Step



# Impedance



# Impedance Close-up



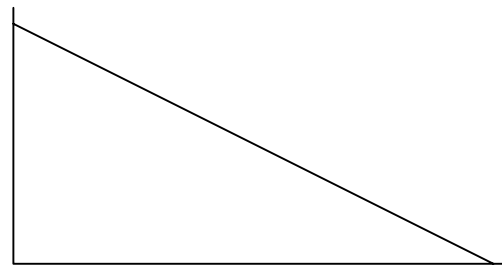
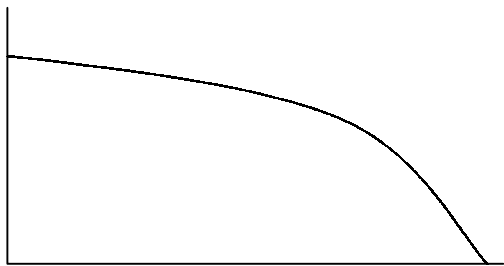
# Saturation

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- Assume linear saturation model
  - Linear is worse than square root

$$\dot{Q} = C \sqrt{P_s - \frac{F_l}{A}}$$

$$\dot{Q} = C \left( P_s - \frac{F_l}{A} \right)$$



# Saturation

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- Assume Case 1: Zero Load Motion

$$F_l = k_s \frac{C \left( P_s - \frac{F_l}{A} \right)}{As}$$

- Saturation Bandwidth

$$\frac{F_{l\_sat}}{P_s A} = \frac{1}{\frac{A^2}{k_s C} s + 1}$$

$$\omega_{sat} = \frac{k_s C}{A^2}$$



# Robot Motion

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- Stability – minimum load mass
- Saturation
- Energy Storage
- Energy Efficiency

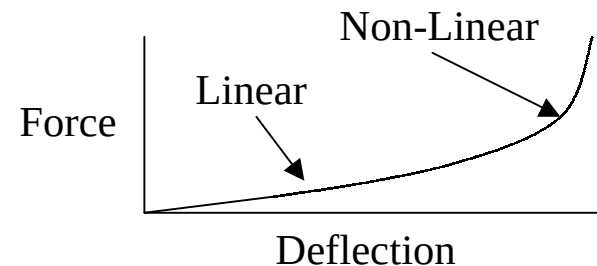
# Non-linear springs

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- Stiffening Springs
  - Low impedance at low  $k_s$ .
  - Increase Saturation bandwidth with high  $k_s$ .
  - Requires state dependent gain control to maintain stability.
  - Follows biology
    - Humans sense a constant percentage change of load force
- Exponential springs
  - Don't exist but we can come close.

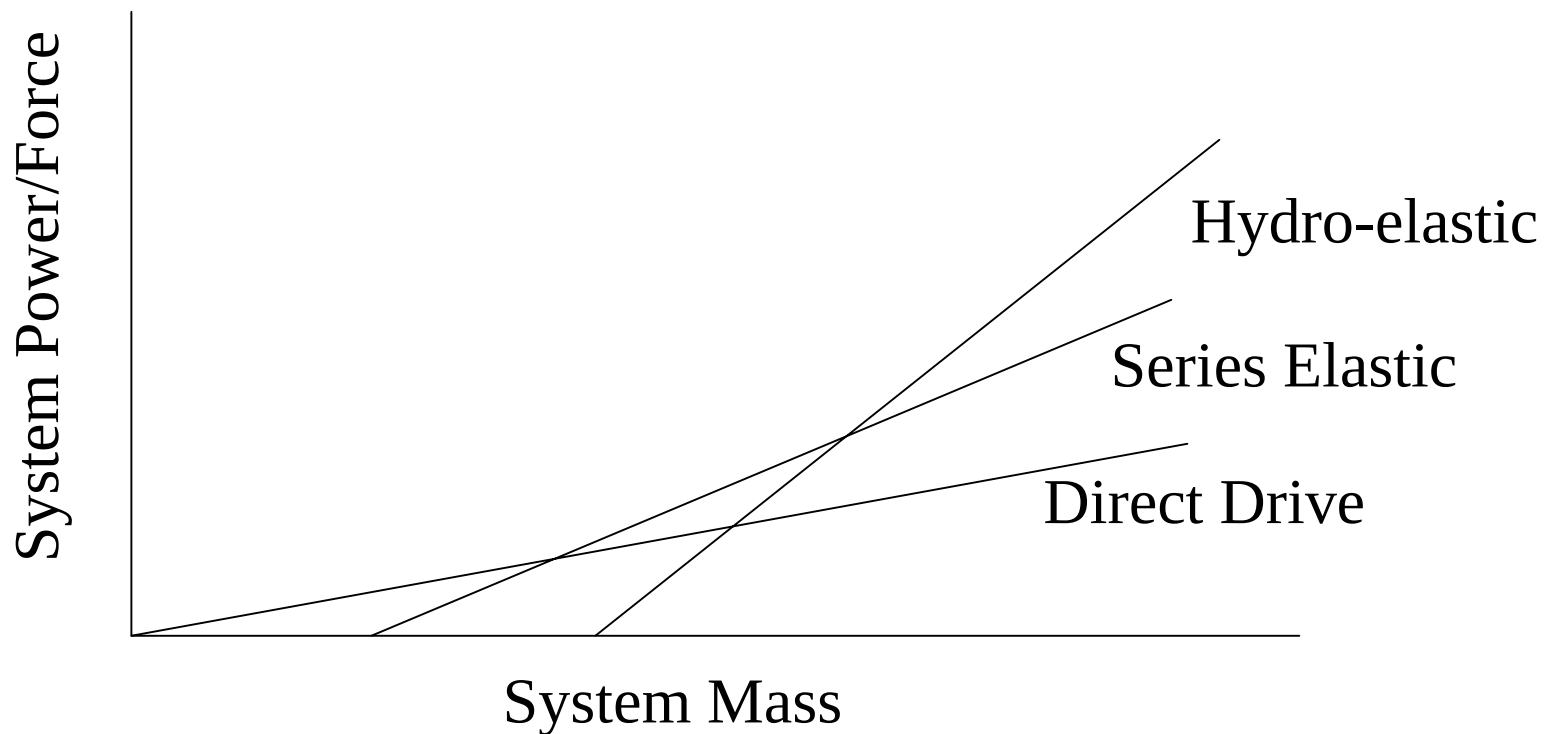
# Non-linear Springs

- Bottoming out springs
  - Volute
  - Tapered coil
  - Good Mathematical Model
- Air Springs
  - Hysteresis due to heat transfer.
- Rubber Springs
  - Low bandwidth
  - Hysteresis due to internal friction



# Force Controllable Actuator Scaling

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# Let's Go Downstairs

- Rm006

