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# Nomenclature

**Force**

$F_d$ — Desired output force.
$F_l$ — Measured output force.
$F_c$ — General model control system output.
$F_e$ — Error between the desired and measured forces.
$F_f$ — Non-linear actuator friction force (i.e. stiction).
$F_{sat}$ — Maximum actuator output force.
$F_o$ — Constant desired output force.

**Motion**

$x_l$ — Motion of the load.
$v_l$ — Velocity of the load.
$x_a$ — Motion of the actuator before the spring.
$v_a$ — Velocity of the actuator before the spring.
$x_b$ — Backlash in the transmission.
$v_{sat}$ — Maximum actuator velocity.

**Power**

$P_l$ — Load power.

**Physical Parameters**

$k_s$ — Actuator spring constant.
$m_m$ — Lumped motor mass.
$b_m$ — Motor friction.
$A$ — Piston Area.

**Frequency**

$\omega_o$ — Saturation frequency.
$\omega_c$ — Controlled natural frequency.

**Gain**

$K$ — Controller gain.
$K_a$ — General model actuator gain.

# Chapter 1

# Introduction

The vast majority of commercial robot applications consist of tasks that control positional trajectory of the robot joints and end point. Traditional robots can do this with great speed, endurance, precision and accuracy. Robots have been used in this commercial niche because repetitive tasks which require precision and accuracy are difficult and tedious for humans.

On the other hand, there are many tasks in which robot competence is inferior to that of biological counterparts. Despite extensive research, actions such as walking, running, swimming, catching, grasping and manipulation, considered easy to most able bodied humans, are difficult for robots. These tasks all require interacting with the real world.

Since our world is essentially an inertial environment, it is important for a robot to be able to sense and control the interaction forces between itself and the environment. This is typically referred to as *Force Control*[1]. There are two aspects to successful force control. The first is to use an algorithm and sensory information to determine what kinds of forces to place within the robot itself and on the robot's environment. The second aspect of successful force control is being able to generate accurate forces at the robot joints and end points. This thesis deals with the second aspect of force control and specifically on the actuators that create the forces.

Present day actuation technology is typically poor at generating and maintaining accurate forces. Using local closed-loop control of actuator forces can improve the quality of force output by rejecting disturbances. Furthermore, it has also been shown that using a significantly compliant load sensor in a configuration called *Series Elasticity*, increases an actuator's ability to generate accurate forces.

The focus of this thesis is to analyze and quantify the effects of significant load sensor compliance for closed-loop actuator force control. There are some unifying principles of series elasticity that can be captured in a general mathematical model. These general principles are further developed with specific models and physical prototypes for the eletro-magnetic and hydraulic power domains. The motivation for this research is driven by a need for guidance in further actuator design and development.

Within this introductory chapter, I define and outline the qualities for good force control actuators. I also describe the first implementations of *Series Elastic Actuators* as a deviation from traditional actuator design. Finally, I define the thesis objectives and give a summary of the thesis results.

## 1.1 Actuation and Force Control

Actuation is the process of taking some form of energy and converting it into a mechanical form of force and motion. An actuator is the device or mechanism which accomplishes this energy conversion process. An electro-magnetic motor with a gear transmission or a hydraulic piston are two examples of actuators.

[1] The term *force* denotes a generalized force/torque vector. The term *position* is the generalized position/orientation vector.

With few exceptions, standard robot actuation systems are very poor at creating accurate forces in robot joints. Some of the reasons for this include friction, stick-slip, breakaway forces on seals, backlash in transmissions, cogging in motors and reflected inertia through a transmission. All of these real phenomena cause force noise in the actuator. The effects of force noise are not quite as noticeable when trying to control the position or trajectory of the robot. The mass in both the robot and the actuators low-pass filter force noise on positional output. This is one reason why robots are so successful at trajectory control even in the face of force noise. However, for tasks requiring good force control, the force noise in the actuators can be problematic.

Good force controlled actuators have several important characteristics which are: force bandwidth, mechanical output impedance, dynamic range, power density and force density. The first three characteristics are related to one another and typically an improvement in one yields an improvement overall. An increase in power or force density usually means that the other three characteristics suffer in performance.

The force **bandwidth** is a measure of how quickly the actuator can generate commanded forces. In other words, to what frequency can the actuator create a one-to-one output force to desired force. The bandwidth must be sufficient to transmit accurate forces through the robot structure. Most force control tasks only require a few hertz bandwidth as witnessed by the bandwidth of human muscle. However, improving the bandwidth of the actuator above the minimal operational level can only improve the overall capabilities of the robot.

Mechanical output **impedance**[2] is defined as the amount of force at an actuator output for a given load motion. In the simplest terms it can be thought of as the stiffness of the actuator output. Ideally, the impedance is zero. Low impedance means that the actuator appears to the robot as a pure force source with negligible internal dynamics. An actuator with low impedance is sometimes referred to as being *backdriveable*.

**Dynamic range** is the maximum output force divided by the minimum output force of the actuator. It gives a measure for how sensitive the actuator is to small forces with respect to its full force output capability. A large dynamic range is desirable because it permits the actuator to be used in a versatile way in both very sensitive activities and in large force situations. Humans have very good dynamic range.

**Force density** and **power density** are not necessarily operational characteristics. Nevertheless, these qualities have reference to an actuators ability to generate force or deliver power to the robot per unit mass of actuator. Having high force and power density means that a small package can output lots of energy. Lightweight actuators prevents over burdening the robot structure with excess mass and allows for quick, responsive robot performance.

Electric motors with gear reductions and hydraulics are two present-day actuator systems that have have high force and power density and can also achieve moderate to high bandwidth. Unfortunately, neither system possess low impedance or large dynamic range qualities required for good force control. Being able to use these systems as force controllable actuators would be very beneficial.

## 1.2 Series Elastic Actuators

The force output capabilities of standard actuator such as an electric motor with gear reduction or a hydraulic piston can be dramatically improved by using a load sensor as feedback for a closed-loop control system. This technique has been used for many years on both actuator closed-loop force control as well as robot endpoint force control.

Standard load sensors are designed to be as stiff as possible for a given load. This allows the sensor to transmit power interactions with no internal storage of energy. Unfortunately, a stiff load cell in series with an already stiff actuator means that the open loop gain of the actuator is very high. This in turn implies that in order to maintain actuator control stability, control gains must be kept low. Low control gains typically indicate poor overall closed loop performance except when operating at very low bandwidths. Low control gains, high impedance actuators and stiff sensors

[2] Throughout this thesis mechanical output impedance is referred to as simply impedance or output impedance.

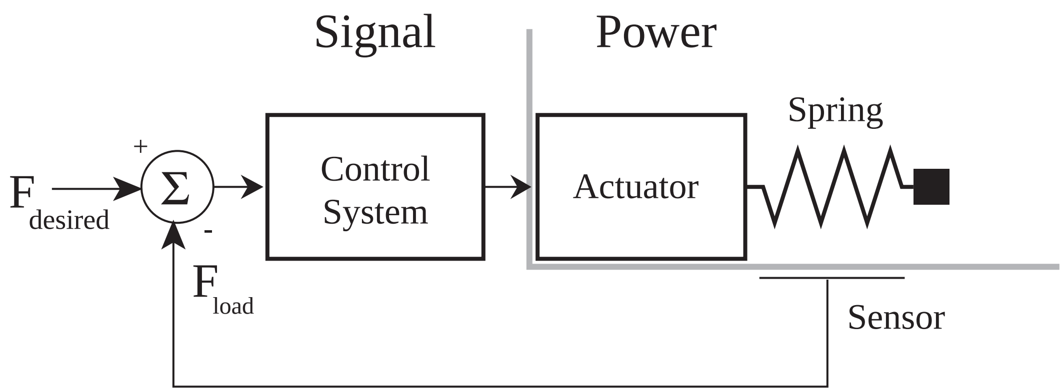


Figure 1-1: A spring is intentionally placed in series with the output of the actuator. The spring deflection, consequently the force, is measured and controlled.

make for a system that cannot handle shock loads from the environment or filter out friction and other transmission non-linearities.

To overcome these deficiencies, several investigators including Howard [28], Pratt [44] and Williamson [68] have experimented with a closed-loop control of an electro-magnetic motor and gear reduction in series with a significantly *compliant spring*. Measuring the relative deflection of the spring or its strain directly results in a load sensor. Pratt and Williamson named this actuator configuration a *Series Elastic Actuator*.

Figure 1-1 shows that topologically, series elastic actuators are similar to any motion actuator with a load sensor and closed-loop control system. However, there are two significant differences resulting from the compliant element. First, there is tremendous energy storage in the spring that cannot be neglected. This can actually be advantagous. Second, integration of a significantly compliant load bearing sensor within a compact actuator can prove challenging. Although difficult, compact design have been sucessfully completed.

The primary advantage of series elasticity is that the compliant load bearing sensor lowers the loop gain of the system and allows for increased controller gain. This allows series elastic actuators to be shock tolerant and have low impedance. In addition, by the increased controller gain, the effects of internal stiction and other transmission non-linearities are greatly reduced.

Many other research and commercial groups have also recognized that purposeful compliance in actuators can be beneficial for force control. Several of these ideas and implementations in both stand-alone actuators and complete robots are described in chapter 2. For the most part, the advantages of compliance in actuators, including series elastic actuators, have been explained with intuitive reasoning and not with mathematical demonstration.

## 1.3 Thesis Objective

This thesis builds and generalizes on the initial work of series elastic actuators. It develops quantitative understanding of the overall effects and requirements of using a significantly compliant load sensor for closed-loop actuator force control. It also resolves issues regarding how performance scales with spring constant and about the tradeoffs for how stiff or how compliant the sensor should be for proper actuator operation. Therefore, the focus of this thesis is threefold.

- I mathematically quantify and analyze the effects of significant load sensor compliance for closed-loop force control.

- I give guidelines in order to select the proper compliance for sensor design.

- I demonstrate the results with experimental hardware.

The thesis objectives are accomplished by developing a general actuator model which has velocity output as its primary causality. This model is representative of current high force and high power density actuator systems which by themselves are not force sensitive. By including a compliant sensor and a simple control system, the general model can help demonstrate and explain many of the properties of series elastic actuators. Specifically the model accounts for controlled actuator bandwidth, output impedance, error rejection, shock tolerance and saturation bandwidth (large force bandwidth). In order to focus on the basic characteristics of the actuator, emphasis is given to simplicity of controller design. Therefore the general model only uses a proportional controller.

The actuator properties in the general model show that there are fundamental design tradeoffs that must be made when choosing a spring stiffness for the sensor element. Discussion of these tradeoffs lead toward guidelines for selection of a proper sensor stiffness. The use of nonlinear springs is discussed in this context.

The actuator properties discussed in the general model are also applied to two case studies for actuators in different power domains. The two power domains are an electro-magnetic motor with a gear reduction and a hydraulic piston. Experimental hardware for the electro-mechanical domain, series elastic actuator, and hydraulic domain, hydro-elastic actuator, are used to verify the models.

## 1.4 Overview of Thesis Results

**Remember to fill this out after completing chapters 3, 4, 5 and 6.**

- Compliant force sensors allow for low impedance, dynamic range, bandwidth
- Compliant force sensors reduce large force bandwidth
- Simple control gets you a long way.
- This has been applied to electric motors with the following results
- Hydraulics with the following results.

## 1.5 Summary of Thesis Contents

The thesis is organized as follows:

**Chapter 1** gives a brief introduction to the background and motivation of the thesis. It also includes a summary of the thesis objectives and covers the key results from the research.

**Chapter 2** presents extensive and detailed background material and related work covering general ideas of force control, present-day actuator technology and actuators which use compliance.

**Chapter 3** describes a genenal model of series elastic actuators. This model contains all of the key characteristics of using a compliant load sensor for closed loop force control.

**Chapter 4** applies results of chapter 3 to a case study of an electro-magnetic motor with a gear reduction. This is the traditional series elastic actuator. The theoretical model is compared with experimental results.

**Chapter 5** applies results of chapter 3 to another case study for a hydraulic piston, a hydro-elastic actuator. The theoretical model is compared with experimental results.

**Chapter 6** gives concluding remarks and outlines a direction for future avenues of investigation.

# Chapter 2

# Background and Related Work

Robots have been very successful at performing tasks that require movement in free space or known environments under position control. Tasks such as spray painting vehicle exteriors, pick and place of IC chips, arc welding and spot welding fit very well into this paradigm. While many robotic applications which use position control have found their way into the commercial sector, robots which contact the surrounding environment and work within kinematic constraints has for the most part been limited to laboratory research. In fact, very simple tasks such as driving a screw, turning a crank, assembling toys, and writing on a chalk board are extremely difficult for most robotic manipulators [5]. Instead of position control, these robots require force control. The robots themselves must be capable of accurately modulating and controlling actuator torques and forces.

This chapter explores the background of force control and force controllable actuators and provides explanations of applicable previous work. Some explanations of different techniques and algorithms to achieve robot compliance are outlined. The bulk of the chapter discusses capabilities of different actuator systems and how each performs with respect to force output control. Actuators with intentional compliance, specifically series elastic actuators, are also discussed with examples of related work in both the electro-magnetic and hydraulic domains.

## 2.1 Force Control

*Force Control* is the term used to describe the necessary interaction between a robot and an external unknown environment. The robot must have the capability of sensing and controlling forces in addition to knowing where it is in its work space.

The necessity for force control deserves some discussion. Paynter gives a general explanation for maintaining proper causality when any two physical systems energetically interact [41]. The two systems' interaction is defined at the interface by a set of orthogonal variables: flow and effort. In a mechanical system, flow is the velocity or motion and effort is the force. In any power domain such as mechanical, hydraulic, electric, thermal etc., there are equivalent flow and effort variables (Table 2.1). The product of these two variables is the power transfered across the system interface. Interestingly, one physical system cannot independently control both the flow and the effort at the point of interface. A system may define and control one of these variables or a relation between the two variables but not both. other.

This explanation of general physical system interaction has direct application for why force control is necessary when a robot interacts with an unknown environment. Most workpieces and environments with which a robot will interact are inertially and kinematically constrained. A pallet has mass. A door swings on a hinge. A wall and floor are vertical and horizontal barriers respectively. Given these constraints the environment often defines the flow interaction. This leaves the robot with only the option to modulate the effort. In other words, while the robot can push on and give energy to the environment, it cannot specify how the environment will respond. If the robot tries to control the position, there exists a basic incompatibility between the two physical systems. Therefore, whenever a robot is not simply moving around in free space and interacts with an environment or

Table 2.1: Flow and effort for different power domains

| Power Domain | Effort | Flow |
|---|---|---|
| Mechanical | Force (N) | Velocity (m/s) |
| Hydraulic | Pressure (Pa) | Flow rate ($m^3/s$) |
| Electric | Voltage (V) | Current (A) |
| Thermal | Temperature (K) | Entropy rate (J/Ks) |

workpiece, it must use force control. While it is still important for the robot to understand its own sense of position, at a minimum level the robot must have some capability of being compliant in order to effectively match the given constraints. As defined in the introduction, the robot must have low impedance. This allows the causality of the robot-environment interface to maintain proper order.

It is important to point out that all environments do have some compliance. While most constraints typically have high stiffness, there are other things which are very compliant such as sod or carpet. In fact some systems, like water, can exhibit variable compliance depending on the conditions. In these cases it may be necessary to control some linear combination of flow and effort. Nevertheless the robot must be capable of being compliant. It cannot be limited to high stiffness only. Therefore, responsive force control is essential.

The following three subsections cover different techniques, ideas and algorithms that robot operators use in order to match compliance requirements with the environment. This includes passive methods as well as active control. The discussion also outlines some of the robot applications where force control can be very useful.

### 2.1.1 Passive Compliance

A simple method to achieve pseudo force control and low robot output impedance is to have a robot use conventional position control but incorporate inherently passive compliance. Passive compliance can be found in the robot structure, joints, and at the end effector. It differs from active force control because passive compliance does not use force information for feedback to modulate the control algorithm. There are two major ideas behind passive compliance: low servo stiffness and passive end effectors.

By introducing passive compliance into the joints, the loop gain for the position control loop is effectively *softened*. A similar effect can be achieved by increasing the compliance in the robot structure (links). With a combination of low servo stiffness and low structural rigidity, the robot may be able to successfully interact with a work piece or environment simply by using position control schemes. However, the risk of adding too much compliance to a robot may be that the robot is *sloppy* in its task execution and fail in its overall goal [16].

Successful examples having compliance in robot joints are the bipedal walking robots build by Honda Research and Development called *P2* and *P3* (figure 2-1). *P3* is a slightly smaller version than *P2*. Both robots use a compliant damping material as a coupling between the robot actuators and joints [17, 23]. The coupling protects the robot's actuators from the shock loads in each step and decouples the reflected inertia through the large Harmonic drive gear reduction. To compensate for *softened* servos, *P3* also has active force sensors in the ankles which give feedback to the control system and are critical to its walking stability.

Passive end effectors are another successful strategy for helping robots interact with the environment. This is because many robots only touch the environment at the manipulator endpoint. A robot can use position control to roughly locate the endpoint properly in its workspace. The endpoint then performs its workpiece interaction task. For example, many end effectors have a soft rubber covering. This effectively decouples the robot from the workpiece through the compliance. Biologic examples of passive compliance at manipulator end points are human hands and feet. The

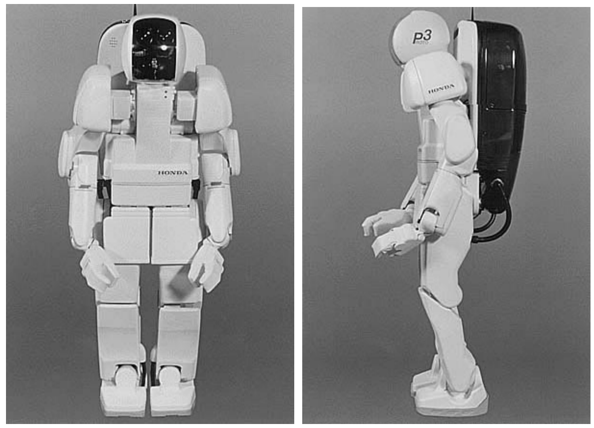


Figure 2-1: *P3*. A commercial walking robot built by Honda Research and Development. It uses a combination of passive compliance in both its actuators and feet as well as active force control at the ankles. Reprinted with permission of Honda Motor Co., LTD.

soft compliant skin compensates for poor positioning accuracy of gross motions and allows for fine manipulation and and also aids shock reduction in locomotion [3].

Interestingly, soft endpoint covering has also been shown to be an effective tool for active force control as well [16]. *P3* (figure 2-1) has a compliant foot in addition to compliance in the actuators. It helps to keep the supporting foot securely in contact with the ground during walking.

Besides a soft rubber covering, pneumatics are also common end effectors that effectively convert the end of the robot into a force control device. This is because pneumatics are inherently compliant. By controlling the pressure in a pneumatic piston, it converts gas pressure into a mechanical force. Pneumatics also maintain low output impedance regardless of the pressure or force at the output.

Another clever passive end-effector is the remote center compliance (RCC) that is set up to automatically perform a peg-in-hole assembly [14]. The configuration of the RCC creates a compliance center at a certain point within its structure. Forces acting on the compliance center of the RCC result in pure translation and torques at that point cause pure rotation. By placing a peg tip right at the compliance center, it can be assembled into it hole without jamming.

### 2.1.2 Active Control

Clearly, passive methods have limits when trying to increase the capability of force controlled robots. Often passive techniques are very task specific like RCC being applied to peg-in-hole assembly. In order for the robot to have more general capability to perform more general tasks, there is a need for active control and task planning. This leads to the need for active feedback of measurements to modulate the control of forces on either the manipulator endpoint, joint actuators or both.

The vast majority of active force control techniques and algorithms have been developed to control and monitor the end point forces and torques via a force sensor at the robot tip. Mason [34] and Whitney [67] give good summaries of formalized compliance control and other active force control methods. Various methods include explicit force control, stiffness control and damping control, impedance control, hybrid position/force control and virtual model control.

Even though this thesis only deals with single degree of freedom force controlled actuators, I assume that a high level force control algorithm is used in the coordination and control of the entire robot system. Therefore, I present an overview of several force control methods and algorithms for multi-degree of freedom robots. Also included are a few general applications which require force control. More will be discussed about the physical capabilities of robots and actuators in the next section.

- **Explicit Force Control** creates a virtual endpoint force *servo*. The controller input is the error between the desired force and actual measured endpoint force. The difficulty in this type of controller is that there is no positional feedback on the robot which means that there is no control on the absolute endpoint position. Therefore, this approach is somewhat difficult to apply to a multi-degree of freedom robot without a higher level control system modulating the forces.

- **Stiffness Control** [55] and **Damping Control** [67] create a virtual six degree of freedom *spring* or *damper* in robot end point coordinates respectively. Both techniques are related because they measures the endpoint forces as well as the joint positions and velocities in order to generate a desired force output relating to the virtual spring or damper. Good torque control on the joints is helpful as the endpoint forces are created with the Jacobian transpose. However, no inverse kinematic calcualtions are needed.

- **Impedance Control** [25] generalizes the ideas of stiffness control and damping control. The position and velocity of the robot endpoint is commanded to follow some trajectory. In addition, the control system modulates a manipulator *disturbance response* which handles the control of dynamic interactions. The robot disturbance response is modeled after a second order mass-spring-damper behavior. In other words, the robot endpoint will behave as if it is a second order system. Therefore the endpoint forces, joint positions and velocities are used to generate actuator torques. The gain matrices which set the effective stiffness, damping and inertia of the manipulator endpoint correlate directly with stability and bandwidth criteria for the robot.

- **Hybrid Position/Force Control** [47] is a method which combines conventional position control and force control. The environment dictates natural constraints such as being in contact with a surface where only force control can be used. Similarly, position control is used in the directions where there are no constrains and the robot can move freely. The trajectory and path that the robot follow are called artificial constraints. A systematic method for understanding and defining natural and artificial constraints has been outlined in several places [5, 35, 60]. Once a task has been defined, actuator torques are then commanded to make the robot operate within the given constraints. This technique requires a new control system for each new task.

- **Virtual Model Control** [45, 46] is a motion control language which allows the control designer to use simulations of virtual mechanical and/or other components, linear or non-linear, to create reaction forces which are transmitted through the robot via joint torques. The robot behaves as if the virtual components are actually attached to the robot. This algorithm method can be formulated into a state machine so that different virtual elements control the robot with each change of state. Virtual model control is different from impedance control in that it does not assume a fixed base and allows component connections between any two points on the robot not just between base and endpoint. While this control language has been applied to legged robots which successfully accomplish their task of locomotion [31], there currently are no formal methods for quantifying stability.

## 2.2 Applications of Force Control

There are many applications which require real time force control. While each application does not have a specific technique associated with it, I list some of the applications to give emphasis to the importance of force control. These applications are: teaching and skill acquisition both for people teaching robots and robots training people, telerobotics, haptics and biomimetic robots.

- **Teaching and Skill Acquisition** [7] – Off-line programming has had only partial success in applications in manufacturing such as grinding and deburring. In these cases, interaction forces for the task must be acquired or learned. These skills are often difficult to teach to people let alone robots. Often it is easiest to train force interaction skill in a teach-by-showing method. It is then possible to embed that information to one of the active control methods listed above such as hybrid force/position control or impedance control. In order for the robot to learn these skills, it must be compliant in the instructors hands. This requires good force control.

Conversely, robots can teach humans. Experiments are being done with robots helping in the rehabilitation process of upper limb control for people recovering from strokes [2]. These robots must be force sensitive and inherently safe.

- **Telerobotics** [56, 57] places a human in the control loop of a robot. One standard form places a human on a master manipulator and the robot slave mimics the motion of the master. To add to the capabilities of the system, a controller will reflect the forces felt by the slave to the master. While it is not recommended that the slave manipulator have endpoint force control from the standpoint of stability, force control locally on each individual joint is highly recommended if not required for proper teleoperation [39]. Advances in manipulator design and control algorithms have allowed for the development of a teleoperated micro-surgery robot [52].

- **Haptics** [53, 51] is closely related to teleoperation where the only difference may be that there is no slave manipulator. A haptic interface will generate forces at its end point as it follows the users motion. The desired forces are created in a virtual environment rather than reflecting a remote real environment. However, the haptic interface gives the illusion that an environment actually exists. Good force control is critical in haptically representing a virtual environment.

- **Biomimetic Robots** are those which try to imitate motions and actions like those of humans and animals. Robots that walk[31], run[48], catch[27], grasp[53], swim[64], etc. are all currently far inferior to their biological counterparts. These robots interact with the surrounding environment and therefore good actuator force control is very beneficial.

## 2.3 Robot Actuators and Active Force Control

The above algorithms and applications, while using different methods of calculation, generate desired output forces for each of the joints in response to joint positions, endpoint forces, desired behavior, and/or the environment. Regardless of the application, algorithm or technique, there is often an assumption made in the control system that high level desired force equals actual force at the robot actuator. For the most part, this assumption is poor, particularly at very low forces when desired force is zero. There are many reasons why this is the case. Friction, stick-slip, intermittent environment contact (impacts), transmission dynamics, actuator dynamics, actuator saturation, reflected inertia and noncolocated control are all limiting factors.

Although actuators are often assumed to be *black box* force devices, the above limiting factors make that assumption invalid.[1] It is important to understand how well various actuation methods generate clean and accurate forces or torques. Studies have been made which compare and contrast different actuation domains including internal combustion engines, electro-magnetics, hydraulics and pneumatics as well as other developing technologies such as shape memory alloys (NiTi), electro-active polymers, braided pneumatics (McKibbon muscles) with biological muscle properties as a benchmark. These studies focused on characterizing and generalizing different actuators into the shape of their flow-effort relationship [21] and on looking at the power density and force density of each actuation method [26].

This thesis only considers the traditional actuation methods of hydraulic and electric actuation. The following discussion focuses on how each one performs with respect to force control. Pneumatics and others actuation methods are mentioned for completion.

[1] It is interesting to note that even many linear and non-linear position control methods and algorithms also assume accurate force generation from the actuators in order to work properly [13, 58].

### 2.3.1 Electro-Mechanical

The vast majority of robotic actuators in use today consist of some form of electro-magnetic motor with a transmission. Electro-magnetic (EM) motors by themselves are well understood. They are fairly easy to model and control because they are for the most part linear. In the simplest model, motor torque is proportional to current. Transmissions, on the other hand, are not linear. They introduce several problems including backlash, flexibility, increased dynamic mass or reflected inertia, and friction.

The purpose for having a transmission is to increase the force and power density of the actuator. This allows the EM motor to run at peak efficiency operating conditions (high speed and low torque) while output power is at low speed and high torques good for robot operation.

While mathematically a transmission works wonderfully, physically it presents problems for both position as well as force control. Non-linearity in the form of backlash, increased dynamic mass and increased output impedance are all a function of the transmission.

Backlash and other non-linearities in the transmission are especially problematic at low forces. The actuator may hunt back and forth in the dead zone between gear teeth in an attempt to modulate zero force. It is possible to create and use non-backlash gears to create a stiffer transmission, however they correspondingly increase drive friction [19].

Power transmissions also increases the equivalent dynamic mass of the motor as seen after the transmission by $N^2$, where $N$ is the reduction. For geared motors, the large reflected inertia leads to broken gear teeth, damaged ball screws and ruined actuators when large unexpected loads appear as the robot interacts with an environment. This is because the large reflected inertia needs time to accelerate in order to store energy. Therefore, the impact energy goes directly into the compliance of the transmission elements until they yield or break. In addition, the motor inertia, as seen through the transmission, will typically be on the same order of magnitude as the robot link inertia and be coupled to the link, significantly altering its dynamic behavior.

Electro-magnetic motors by themselves are typically very easy to backdrive and have low impedance. This means that a small external torque on the motor shaft will cause it rotate. However, with a large transmission, due to the increased real and apparent friction and increased reflected inertia, an electro-magnetic motor becomes significantly more difficult to backdrive. With this increased impedance, the actuator becomes much more of a position causal system rather than a force causal one.

To overcome the difficulties associated with linking an electro-magnetic motor to a transmission, many novel approaches have been tried. Three of these approaches are described below which include: direct-drive, cable transmissions and evoloid gear sets.

In an effort to make a clean, low-friction, low-impedance actuator, Asada et. al. created direct-drive actuators and robots [6]. Direct-drive eliminates the transmission and connects a DC brushless motor directly to a robot link. This construction eliminates friction and backlash and creates a very force sensitive actuator. Advanced torque sensors in the actuators have added to the capabilities of direct-drive robots [1].

From a modeling and control standpoint, direct-drive robots are simple systems. The motor (magnet) mass is included with the link inertia. Unfortunately, to compensate for the loss of transmission, direct-drive motors must be large in order to achieve high torques. High torque requires high current which in turn requires heavy wiring. In weight sensitive and power sensitive robot applications such as mobile robots, direct-drive actuators are often unacceptable. Nevertheless, for ground based robots with few degrees of freedom, direct-drive actuators have been very successful in both force control applications and position control, particularly in semiconductor manufacturing automation.

In order to improve the force and power density of EM motors without sacrificing force sensitivity, work has also been done to create stiff, low-friction, *clean*, light-weight cable transmissions [63]. These transmissions have zero backlash and high power efficency due to high tensioning in the cables. However, because of the size constraints of pullies, cable transmission can only achieve moderate transmission ratios. Nevertheless, cable transmissions have been used on novel robots such as the *Whole Arm Manipulator* [54] used in robot catching [27] and digging, the *PHANToM* [36] used as a three degree of freedom haptic point interface, *Robotuna* used to study undersea oscillating foil propulsion [8, 64] and robots for minimally invasive telerobotic surgery [33, 52].

Force control with slightly higher ratio gearing has been achieved with very careful actuator design and development in the construction of *Artisan*, a ten degree-of-freedom manipulator [65]. Instead of using single stage helical gears, Vischer and Khatib used *evoloid* [9] gear sets for each joint. These gears have higher load capabilities than helical gears but are still backdrivable. This transmission used in conjunction with a novel torque sensor, has helped *Artisan* to demonstrate compliant behavior.

### 2.3.2 Hydraulic

In contrast to electro-magnetic motors, hydraulic systems operate best at high force (high pressure) and low speed (flow rate) which is ideal for robotics. Hydraulic systems also have one of the highest power density of modern controllable actuation methods [26]. Systems often operate at 3000 psi and higher. Hydraulics can hold large loads indefinitely while consuming minimal power. This is something that would cause electro-magnetic systems to quickly overheat. Hydraulics are particularly useful in high force and high power density situations such as construction machinery, airplanes and automobile steering systems. Nevertheless, there are several downsides to hydraulics which have caused electro-magnetic servos to be favored in the robotics industry. These include potential messiness, system complexity, high impedance and non-linearity from a control perspective.

There is an inherent messiness with hydraulics especially when hydraulic oil is used. This is especially true for catastrophic system failure such as a tree falling onto hydraulic forestry equipment [38]. Modern system design has alleviated some concerns regarding leakage of hydraulic fluid but the threat is always there.

Since pressure and return lines must be run to hydraulic actuators, there is increased system complexity from a plumbing perspective. The hoses and pipes going to and from the actuator are non-trivial in size. They must be accounted for in robot design and can potentially add significant mass to the robot structure.

Hydraulic systems also have high impedance. This is a result of the incompressiblity of hydraulic fluid and the fact that servo valves control fluid flow. All fluids are compressible to some extent however traditional hydraulic fluids, such as oil or water, are typically modelled as incompressible. The only way to backdrive (backflow) a hydraulic system is to create a load pressure higher than the source pressure. In this case, fluid reverse flow because flow in an open servo valve is a function of pressure diffenence. Hydraulics are specifically designed for motion control, not force control.

The final downside to hydraulic systems is that they are very non-linear from a control perspective. Null bias, null shift, hysteresis, threshold, internal leakage, square root flow-load characteristics and unequal piston areas are all non-linear effects in hydraulics. Depending upon the level of precision required by the robot, some or all off these effects need to be accounted for.

The advantages of hydraulics often outweigh the disadvantages and are therefore used quite frequently in position controlling robots and heavy equipment. Hydraulic actuators typically consist of a pressure source and a flow control valve (i.e. spool valve). A small signal current deflects the spool valve which allows the high pressure fluid to flow. In the simplest linear model, the fluid flow rate through the valve is proportional to the small input current. The valve directs high pressure fluid into one of two chambers which drives a piston thus converting fluid flow into a mechanical motion. Thus, in this configuration hydraulics are very good at position control.

It may seem natural that it would be possible to modulate the piston chamber pressure in order to get good force control. Unfortunately, the pressure in the chamber is not a good representation of the force at the actuator output. A few reasons for this include: friction and stiction in the piston and seals, supply pressure variations and non-linear flow characteristics, and high output impedance. The friction and non-linearity create force noise on the actuator output. High impedance comes from difficultly in back-flowing through a servo valve as well as the fact that hydraulic fluid has great inertia. In general, force control of hydraulic actuators is a difficult problem [12].

There have been attempts to overcome some of the difficulties associated with hydraulics for use in robotic contact tasks. These include mechanically decreasing the seal tolerance of the piston and implementing advanced control algorithms.

In order to reduce the sliding friction and stiction, the seal tolerance can be lifted. But, that has a direct effect on system efficiency not to mention the problems of keeping the work environment clean. In Raibert's hopping robots, the legs had two sets of loose seals [48]. The first set allowed leakage flow from the pressure chambers. The second seals simply scavenged the leakage. This configuration helped the piston to slide very smoothly. Even so, using pressure as an estimate of actuator output force proved inaccurate.

Given the natural non-linearity of hydraulic systems, a non-linear Lyapunov based controller has been applied to hydraulic force control [4]. This controller also includes adaptive behavior for parametric uncertainty compensation since many parameters in the hydraulic system are either difficult to measure off-line or are time varying with temperature.

Hydraulic impedance control has been implemented on an industrial hydraulic robot. The algorithm for control is a specialized one which recognizes that many robot actuators are position causal and is called position-based end-point impedance control [42]. While the implementation was somewhat successful, due to limitations in the positioning accuracy and the bandwidth of actuators, the robot would exhibit a hunting behavior when in contact with a semi-stiff environment [22].

It is very interesting to note that the hopping robots and the Lyapunovo controller also included springs in their actuators. This effect of the compliance will be discussed in the section 2.4.

### 2.3.3 Pneumatic

Pneumatics typically consist of a piston and a valve connected to a gaseous pressure source. Pressure in a piston chamber converts the piston into a force compliant actuator. Pneumatic actuators have many features which are close to hydraulics but with significant differences resulting from the operating fluid. Inherent compliance from the compressibility of gas make these useful as end effectors. Clever design configurations and control schemes have demonstrated that it is possible achieve moderate positional bandwidths of 21-35 Hz [26].

There are several problems with pneumatics that limit it's applicability to robotics including safety, thermodynamic effects and potential resonance. First, unlike hydraulics, pneumatic systems run at relatively low pressures (100 psi) for safety reasons. They can operate at higher levels but compressed air stores vast amounts of energy. If there is a rupture in the line, pneumatics become very dangerous. Thermodynamic effects of compressing and expanding air can heat and cool the system dramatically. Valves and seals can be designed to operate within temperature variations; however, it is usually not recommended. Finally, the natural compliance of pneumatics can also resonate with robot link inertias. Clever damping schemes are often required to maintain stability.

A variation on the standard configuration above is an inflatable elastic tube covered by a flexible braided mesh typically called McKibben muscles [26]. When pressurized, the elastic tube inside expands but is constrained by the mesh. The flexible mesh shortens or contracts like a muscle due to the expanding tube. It has been found that McKibben muscles can exhibit passive behavior very similar to biological muscle [32] since it has both series and parallel elasticity. These actuators have been shown to also have force and power density similar to electro-magnetic systems.

### 2.3.4 Others

There are many emerging actuator technologies with potential for use in macro scale robots such as shape memory alloy (NiTi), electro active polymers, polymer gels, piezoelectric and dielectric elastomers [10, 26, 43]. Some of these systems demonstrate moderate force density and look very promising. However, there are some technical hurdles that must still be overcome. These include long actuation time, small scale, low power density and low efficiency.

Several of the systems have very long actuation time constants. This is the case for polymer gels as well as shape memory alloy. Specifically for shape memory alloy, actuation time is thermally limited. The elongation time is fairly quick because high current can be pulsed through the actuator. However, in order for it to return to its initial shape, the latent heat in the actuator must be removed which can take a long time.

For the most part, electro active polymers, piezoelectric actuators and dielectric elastomers have current working prototype systems on a very small scale. This is ideal for actuation of micro robots. In order to work on macro robots, however, work must be done to scale the systems in size.

The final difficulty with many of these actuators is that they have low output power density and are very inefficient. The power density is orders of magnitude lower than electro-magnetic motors and even lower for hydraulics. Also, the input power to these systems is still quite high which gives a very poor efficiency rating. Hopefully future development of these systems will make them more competitive with current macro scale robot actuators.

## 2.4 Intentionally Compliant Robot Actuators

Each actuation method described above has strengths and weaknesses with respect to force control, force density and power density. The predominant theme for the actuators is that those with high force and high power density typically have high impedance. Therefore, as is, these actuators are difficult to use in force control situations regardless of sensory feedback information.

In an attempt to overcome this impasse, it is possible to decouple the dynamics of the actuator from that of the robot by placing a compliant element between the two. This gives passive compliance to the actuators. However, by measuring the deflection of the spring, an estimate of force in the joint is obtained and can be used for active feedback control.

Introducing compliance into the drive system is contrary to conventional machine design wisdom. Traditionally, machines and drive systems are built to be as stiff as possible to increase bandwidth. However, it is interesting to note that the actuator benchmark for force control applications, biological muscle, is connected in series to a link output through an elastic tendon [3]. There are many reasons why this is thought to be the case. Ideas include energy storage and increased efficiency, stability when contacting environments and filtering shock loads to the body. Even though muscles do not have any of the same problems inherent to EM motors and hydraulics, elastic elements can be useful to actuation regardless of being biological or technological.

Actively measuring a compliant element in an actuator was first applied to electro-magnetic motors with a transmission and called *Series Elastic Actuators*. This section gives a history of these actuators as well as other systems which use active compliance.

### 2.4.1 Series Elastic Actuators

Howard [28] first built an actuator with a spring in series with the motor and transmission. He then measured the relative spring deflection by taking the difference between the motor rotation and output shaft rotation. By controlling this deflection he essentially was controlling the actuator output torque by Hooke's Law:

$$F = kx. \tag{2.1}$$

Unfortunately, using a differential measurement between motor and joint misses the actual strain in the spring due to transmission compliance and is also noise prone. Nevertheless, he was able to show that by using a smaller motor and larger transmission ratio, the output force and power density increased while maintaining good control over force.

Building further on the ideas that elasticity in an actuator could be beneficial, Pratt[44] and Williamson [68] created a similar actuator to that of Howard's. They also showed that to get a better force measurement for closed-loop control, the strain/deflection of the spring should be measured directly and not as the difference of the motor and output shaft. This actuator configuration is called a *Series Elastic Actuator* (figure 2-2). Like Howard's actuator, the series elastic actuator mechanically filters friction and backlash and other non idealities in high ratio transmissions drives. It also filter shock loading from the environment.

In addition to initial prototypes, versions of series elastic actuators have been demonstrated in a few robots.

- *COG* (figure 2-3) [11] is a humanoid robot with with upper torso and head. Series elastic actuators are used in its arms which have 6 degrees of freedom. Using the natural dynamics of

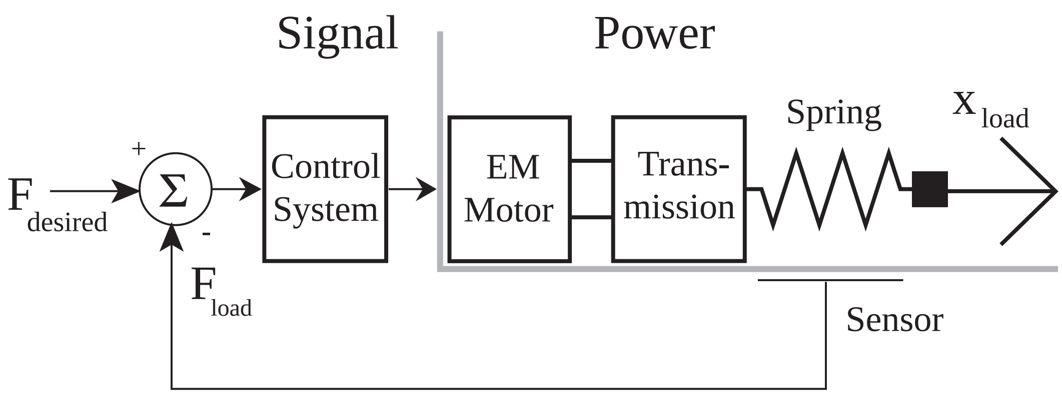


Figure 2-2: Series Elastic Actuator. A spring is intentionally placed in series with an eletro-magnetic motor, transmission and actuator output. The spring deflection is controlled thus inferring force control.

the arms, low noise force controllable actuators and dynamic oscillators for control, *COG* was able to do many tasks previously thought to be very difficult for robots such as hammering and turning a crank [69].

- *Spring Flamingo* (figure 2-4 is a planar bipedal walking robot with series elastic actuators driving 6 degrees of freedom (three in each leg for hip, knee and ankle) [31]. It can walk at $1m/s$ on flat terrain. It has also walked blindly over terrain of alternating 15 degree slopes.

Series elastic actuators work successfully over a moderate range of spring stiffnesses. Unfortunately neither Howard, Pratt nor Williamson discussed the tradeoffs of various spring stiffnesses nor did they give design guidelines for choosing an appropriate spring constant in the actuator. The actuators in the robots were mostly designed through many trial and error iterations. This thesis provides a more quantifiable understanding of the effects of the springs in the system and also provides direction to spring selection in actuator design. Some of this work has been reported previously as a consequence of further robot development [50].

### 2.4.2 Other Electro-mechanical Compliance

There are other examples where springs have been used in series with transmissions. Elastic transmissions are an integral part of Morrell's parallel coupled micro-macro actuators [37]. In this scheme, a small direct drive actuator is coupled in parallel with a larger geared motor through an elastic transmission. The large motor creates a force with gross sensitivity and the small direct drive motor accounts for the error. While increasing the complexity of the actuator, a substantial increase in dynamic range is achieved.

In the control and handling of arbitrary two dimensional objects, Hanafusa created a three finger gripper with springs at the end of each finger [20]. The fingers were driven by a single motor and by measuring the deflection of each spring, stable grasping forces were achieved.

In the manipulator design for mobile robots to do cooperative object handling, Sugar built a three degree-of-freedom parallel manipulator which can generate forces in the $x$ and $y$ directions and a moment in the $z$ direction [62]. The actuators for the manipulator have springs after the motor and transmission and the force is controlled by measuring the deflection of the spring. This design is very similar to series elastic actuators described above.

Sugano designed and build a compliant one degree-of-freedom finger [61]. The finger's compliance could be set by changing the effective length of a leaf spring which was in series with the motor and transmission. The actuator required three motors. One would drive the position and one would set

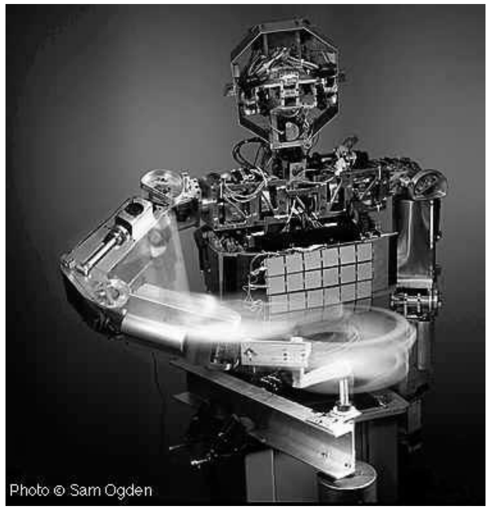


Figure 2-3: COG is a humanoid robot with upper torso, arms and head. The arms use series elastic actuators for each of its six degrees of freedom. Here COG is turning a crank using a dynamic oscillating controller.

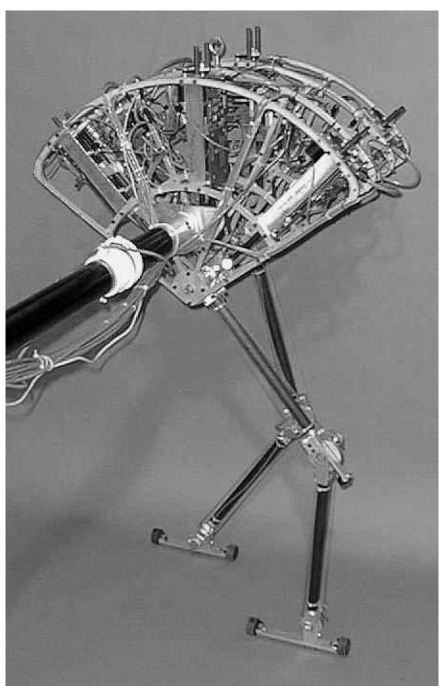

Figure 2-4: Spring Flamingo is a planar bipedal walking robot. It uses series elastic actuators to actuate its six joints. Its top walking speed is 1 m/s.


Figure 2-5: Sarcos Dextrous Arm. It is a hydraulic force controlled teleoperated manipulater that has been used for many applications including production assembly, undersea manipulation and hazardous material handling. The manipulator actuators have accumulators on either side of the hydraulic fluid chambers. This gives the actuators intrinsic compliance. Reprinted by permission of Sarcos.

the spring stiffness. An additional motor was required as a brake and used to damp out the residual vibration from the spring.

### 2.4.3 Hydraulic Compliance

Springs and other compliant mechanisms have also been used in a similar manner for hydraulic actuators. Following the example from biologic understanding of running [3], Raibert's hydraulic legged robots use springs to recover impact energy from each step during locomotion [48]. Using the stored energy in the springs during stance, these same robots were able to perform simple gymnastics such as a flip [24]. These robots were not under force control; however, the springs in the legs are necessary to maintain stability under high shock loading with each step.

As discussed previously, Alleyne designed and demonstrated a Lyapunov based controller for hydraulic piston force control [4]. The piston had a standard load cell attached to its output. However, rather than using an inertial load, the force sensor pressed on a spring fixed to ground. Interestingly, the spring compliance dominated over the sensor compliance and essentially lowered the loop gain and allowed the actuator to perform as well as it did.

The first force reflective teleoperated hydraulic manipulator was built by Sarcos, Inc. (Salt Lake City, UT) [29] (figure 2-5. Using one of the actuators for this robot, Wells characterized a hydraulic piston with accumulators which gives the actuator intrinsic compliance [66]. Instead of a mechanical spring at the actuator output, the piston has a fluid spring on the inside. Wells demonstrated that due to the compliance in the piston chambers, the actuator was much better able to tolerate external disturbances.

As with electro-mechanical series elastic actuators, little published work has been done to quantify the effects of compliance for hydraulic actuators. This thesis applies the ideas of series elastic actuators to the hydraulic domain. Similar ideas about the spring design tradeoffs and considerations are discussed. The initial work on this subject has been reported previously [49].

## 2.5 Summary

There are many applications for robotic force control, especially when the robot needs to interact with an external unknown environment. While there exists several types of algorithms and methods for implementing force control, there is a lack of actual robots than can achieve expected performance. One primary reason for this limitation is the robot actuators.

Robot actuators are especially poor at force control when high force and power density are required. In order to achieve the high power density, electro-magnetic motors require power transmission which introduces many problems including increased friction, large reflected inertia and high impedance. Hydraulic actuators achieve vhigh force and power density requirements. However, force at the end of the hydraulic piston is difficult to control due to non-linearities, pressure variation and seal friction. Hydraulics, along with electro-mechanical actuators, are much better suited for position control.

Closed-loop actuator force control has been shown to be improved with the introduction of compliance and controlling the deflection of the compliant member. The rest of the thesis describes and quantifies this finding. I create a general theory for series elastic actuators and give two case studies for implementing a series elastic design in the electro-mechanical and hydro-mechanical domains.

# Chapter 3

# Linear Series Elastic Actuators

As discussed previously, this thesis focuses on understanding the effects of placing a significantly compliant spring in series between an actuator output and load. By measuring the compression of the spring, the force force in the spring and thus the force on the load can be inferred. Closed-loop feedback control then modulates the actuator system to obtain the desired output force. This technique has been termed *series elastic actuation.*

This chapter discusses the fundamental principles of series elasticity using a general model which is not power domain specific. The general model demonstrates closed-loop bandwidth, closed-loop output impedance, internal error rejection, impact tolerance and large force saturation bandwidth. This chapter provides a unifying framework for better understanding series elastic actuators as applied to two specific case studies in the following chapters.

## 3.1 General Model

As discussed in the previous chapter there are many methods of actuation each with their own set of desirable and detrimental characteristics. Typically, actuators with high force and high power density have poor output impedance and are better at position or velocity output.

The main idea behind series elasticity is to take a standard actuation method with velocity output, add compliance at the end point and use feedback control to modulate the force acting through the spring. This is done by directly measuring spring deflection or strain in the spring.

Figure 3-1 shows the main components of a series elastic actuator with the shaded line indicating the division between power domain and signal domain. The main elements are:

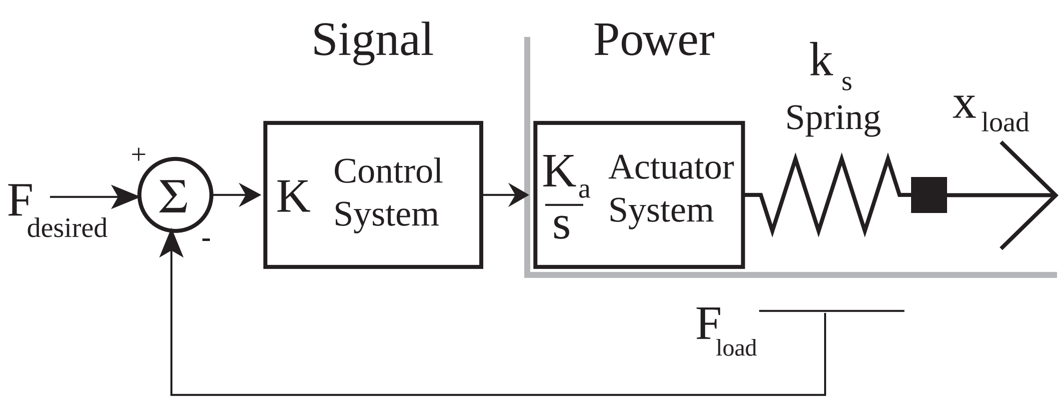


Figure 3-1: General model for series elastic actuators.

- System Inputs
- Actuator System
- Compliant Sensor
- Control System

It is important to point out that topologically, this generic model is no different than any actuator with closed-loop force control. Any actuator of this sort will have an actuator system, a sensor with compliance and a control system. The major difference between series elasticity and other actuators with closed-loop force control lies in stiffness of the sensor. This subtle yet very important difference is the key to series elastic actuators.

In section 3.1.1 through section 3.1.4, I define and describe the four main elements of the general model for series elastic actuators. The definitions lead to the derivation the open-loop model and subsequently the closed-loop general model for series elastic actuators in section 3.1.5.

### 3.1.1 System Inputs

There are two system inputs:

- $F_d$ – desired output force.
- $x_l$ – motion from the load or robot link.

The desired force is in the signal domain. The value of $F_d$ is generated by a higher level controller. Ideally, the output force of the actuator equals $F_d$ with one-to-one correspondence in magnitude and frequency. Even though output of the actuator has limitations, to some degree the actuator can be considered a *black box* force device.

The motion of the load is in the power domain. The actuator will typically be connected to an inertial robot link which defines the motion, $x_l$. This motion constraint on the actuator output is discussed in section 2.1 in the previous chapter. It has also been explained by several other authors in terms of admittance/impedance [25] and position causality of the environment [30, 41, 59].

### 3.1.2 Actuator System

The actuator system converts energy into mechanical force and motion. For the electro-mechanical domain, the actuator system consists of a motor, transmission and motor amplifier. Force on the motor mass creates a velocity output through the transmission. Similarly, for the hydraulic domain, the actuator system consists of a piston and servo valve connected to supply and return pressure. The servo valve modulates fluid flow and thus velocity of the piston output.

For the purpose of the general model, the actuator is assumed to have a simple transfer function relating the input force $F_c$ from the controller to the output velocity of the actuator $v_a$.

$$v_a = K_a F_c. \tag{3.1}$$

$K_a$ is the gain of the actuator relating force to velocity. It is assumed that the actuator can perfectly generate the commanded force $F_c$ and therefore creates output velocity.

The force in the spring is a function of position. Using the fact that in the Laplace domain, actuator position is related to velocity by

$$v_a = x_a s. \tag{3.2}$$

Substituting equation 3.2 into equation 3.1 and solving for the actuator position output $x_a$, creates a transfer function relating commanded force to position written as

$$x_a = \frac{K_a}{s} F_c. \tag{3.3}$$

Such a simple model is hardly representative of real actuator systems and assumptions to justify this model are extreme. Nevertheless, a first order actuator model very clearly captures the essence of series elasticity. More complex actuator models will be applied when discussing the case studies of two real actuators in the following chapters.

### 3.1.3 Compliant Sensor

The sensor is a transducer and thus has two parts. One part, the spring, is in the power domain and the other part, the strain sensor, is in the signal domain.

The spring is load bearing but passive. It is placed at the output of the actuator system. The spring has a spring constant of $k_s$. Depending on the spring stiffness, the deflection can be significant in order to cover the full force output range of the actuator.

The sensor measures the deflection or strain in the spring which is a representation of the force, $F_l$, acting through the spring. The relationship between force and spring deflection is defined by Hooke's Law [18]:

$$F_l = k_s x_s \tag{3.4}$$

where $x_s$ is the spring deflection.

In an ideal world $x_s$ is defined by the difference between the actuator position $x_a$ and the load position $x_l$.

$$x_s = x_a - x_l. \tag{3.5}$$

While it is possible to measure both $x_a$ and $x_l$ and take the difference for the measurement, it is not recommended. There is uncorrelated noise between these two measurements due to transmission dynamics and other non-linearities. In order to reduce noise from the compliant sensor, it is much better to take a direct measurement of the spring strain $x_s$.

### 3.1.4 Control System

The spring strain measurement $x_s$ implies the load force $F_l$ (equation 3.4) and is used for feedback to a control system. Given that there is some desired force $F_d$ for the actuator, the the error between these two signals will define the input to the controller $F_e$:

$$F_e = F_d - F_l. \tag{3.6}$$

Throughout this thesis, emphasis is placed on controller simplicity in order to understand the basic characteristics of the actuator. For the purpose of this discussion, the control system is represented by the controller gain $K$. Therefore, the control law for the general series elastic model is:

$$F_c = K F_e \tag{3.7}$$

I recognize that with further controller development that improvements in overall system performance are often realized. However, a proportional control law $K$ clearly demonstrates the effects of the compliant load sensor. Prematurely introducing complex control systems cloud understanding of the underlying issues. There are ample methods for controller design once the basic concepts of the plant characteristics are clarified [40, 58].

### 3.1.5 General Linear Model Derivation

With each of the main elements of a series elastic actuator defined, the general model can now be derived. Figure 3-2 is just a simplification of figure 3-1 for clarity. The goal is to control the force in the spring $F_l$. I use the model in figure 3-2 to derive the linear dynamic equation for the force in the spring, $F_l$, as a function of the two inputs $F_d$ (desired input force) and $x_l$ (position of the load).

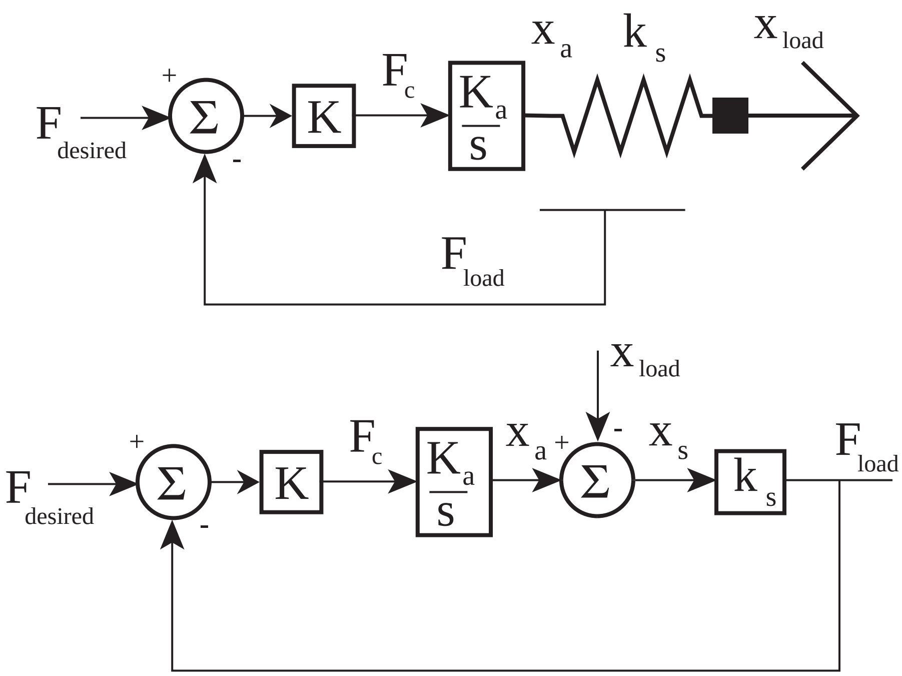


Figure 3-2: General model and block diagram for a series elastic actuator. This model helps to define the open-loop and closed-loop transfer functions.

### General Power Domain Open-Loop Model

The force in the spring $F_l$ is directly proportional to the spring's displacement $x_s$ (equation 3.4). $x_s$ is a function of the actuator system output $x_a$ and the motion input constraint $x_l$ (equation 3.5). Substituting equation 3.5 into equation 3.4 for $x_s$

$$\begin{aligned} F_l &= k_s x_s \\ &= k_s \left(x_a - x_l\right) \end{aligned}$$

and subsequently putting equation 3.3 into the result for $x_a$ yields the open-loop dynamics for the force in the spring of a general series elastic actuator:

$$\boxed{F_l = k_s \left(\frac{K_a}{s} F_c - x_l\right).} \tag{3.8}$$

Equation 3.8 shows that the open-loop power domain model is a function of the input force from the controller and the motion of the load.

### General Closed-Loop Model

The closed-loop dynamic equation can be derived by including feedback and a control law. From equation 3.7, the input force to the actuator $F_c$ is simply proportional to the error difference from the desired force.

$$F_c = K\left(F_d - F_l\right) \tag{3.9}$$

Combining the open-loop power domain model (equation 3.8) with the control law (equation 3.9) gives

$$
\begin{aligned}
F_l &= k_s\left(\frac{K_a}{s}F_c - x_l\right) \\
&= k_s\left(\frac{K_a}{s}K(F_d - F_l) - x_l\right).
\end{aligned}
$$

Solving the equation for $F_l$, we get the closed-loop dynamic equation of a general series elastic actuator

$$
\boxed{F_l = \frac{k_sKK_aF_d - k_s s x_l}{s + k_sKK_a}.} \tag{3.10}
$$

The inputs for equation 3.10 are as described previously in section 3.1.1, the desired force $F_d$ and the motion of the load $x_l$.

## 3.2 Actuator Characteristics

Remember that open-loop model (equation 3.8) and the closed-loop model (equation 3.10) are symbolic and do not represent a physical actuator system. This generic model does however, help us look at the key characteristics of series elastic actuators. In particular, the open-loop power domain model of equation 3.8 and the closed-loop model of equation 3.10 help us discuss:

- Closed-Loop Bandwidth
- Output Impedance
- Error Rejection
- Impact Tolerance
- Saturation Effects

By looking at each of the two inputs of equation 3.10 independently, the closed-loop bandwidth and output impedance can be understood as two separate cases. These two cases completely define the linear characteristics of the actuator.

The first case has a fixed load where $x_l$ is constant. This is the closed-loop forward transfer function. This shows the closed-loop bandwidth as well as the bandwidth profile. The relative values of the control gain, $K$, and the spring stiffness of the sensor, $k_s$, play an important role in defining the actuator bandwidth.

In the second case, the load is free to move and the desired force is held constant. This is the output impedance which is a measure of how well the system responds to external disturbances. In other words, how well are internal dynamics of the actuator decoupled from the load. I explain the important role that the the spring stiffness, $k_s$, plays in minimizing output impedance.

There are internal non-linearities such as stiction and backlash within the actuator system that hinder performance. Closed-loop force output control with a compliant sensor reduces the magnitude of these errors. Error rejection is explained in terms of the magnitude of the ratio of $K$ and $k_s$.

Impact tolerance has to do with the ability of the actuator to handle impact power from the environment. This is a function of the external motion constraint in the general model and the output impedance described in section 3.2.2. A soft spring stiffness in the sensor and an increased controller gain help the actuator store energy and minimize peak impact power.

Saturation is solely a function of the power domain and is not explicitly included in the linear block diagram model. In the power domain, power transfer in the system bilaterally affects connected lumped elements. In other words, for the model in figure 3-1, the force and velocity in the spring affects the actuator systems output capability. Therefore with the addition of a simple saturation limit on actuator output, the effects of a soft spring stiffness, $k_s$, on overall system performance can be discussed.

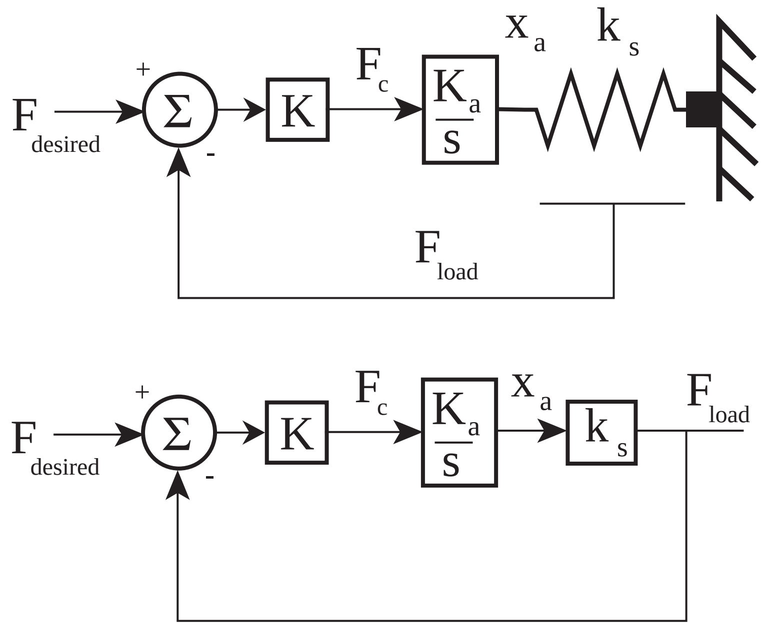


Figure 3-3: Fixed Load Model and Block Diagram. The closed loop dynamic response is clarified with a fixed endpoint constraint.

### 3.2.1 Case 1: Fixed Load – Closed-loop Bandwidth

By assuming a fixed load, $x_l = 0$ (figure 3-3), we can write the closed-loop forward transfer function relating the load force $F_l$ to desired force $F_d$.

$$G(s) = \frac{F_l}{F_d} = \frac{k_s K K_a}{s + k_s K K_a}. \tag{3.11}$$

Clearly, at low frequency, the transfer function is unity. There is a one to one correspondence to the desired force and measured output force. At some frequency the actuator response begins to drop off (bandwidth). Since this is a first order model, the controlled bandwidth, $w_c$, can be defined by:

$$\omega_c = k_s K K_a \tag{3.12}$$

It is a function of the spring constant, $k_s$, controller gain, $K$, and actuator gain, $K_a$.

In this simple case there seems to be no limit to controlled actuator bandwidth. Increasing the gain $K$ will apparently give any desired bandwidth. However, practical intuition says that actuators cannot have infinite bandwidth.

Work in understanding the fundamental limits to force control bandwidth has been done previously [15, 16]. This previous work looks at the problem of robot endpoint force control. In order to maintain clarity, the analysis in that research is simplified to focus on a one degree-of-freedom robot. The results of that research directly apply to this thesis which focuses on a one degree-of-freedom actuator. Topologically, the two models are equivalent.

Since this work has been done previously, I do not perform an analysis of force controlled actuator bandwidth limitations. For the sake of completion, I list the findings of the previous work which gives several potential bandwidth limits. The fundamental the bandwidth limits applicable to the actuators are five fold:

- Non-colocated dynamics – Dynamics between the the power modulator (i.e. motor amplifier or servo valve and the sensor.

- Plant nonlinearities (i.e. transmission backlash and friction, actuator saturation, intermittent contacts such as impacts)

- Bandwidth limitations of the power modulator.

- Finite sensor resolution

- Controller Dynamics

Depending on the design of the actuator system, any of these five can be the limiting factor. I use the analysis in the previous work and discuss how each limitation applies to series elastic actuators.

Physical examples of non-colocated dynamics include compliance in the transmission of electro-mechanical actuators and compressibility of the fluid in hydraulic actuators. Non-colocated dynamics introduce phase into the system. If the actuators operate above the frequency range of the these dynamics, it will drive the system to instability.

For most actuator systems, the plant non-linearities, such as stiction or backlash in the transmission or intermittent contacts, hinder the ability to effectively modulate closed-loop force control. Control systems often cause these actuators to hunt around the desired force and cause chatter. To eliminate hunting and chatter, control gains are reduced which in turn reduces the controlled bandwidth. It is interesting to note that when using a stiff load sensor, the magnitude of backlash in a transmission can represent a significant portion of the sensor deflection. As is shown later, using a compliant sensor reduces the effect of backlash and other non-linearities.

Even though the power modulator in an actuator system is often assumed to have infinite bandwidth, it too has bandwidth limitations. A motor amplifier for the electro-mechanical domain and a servo valve for the hydraulic are physical examples of the power modulator. The motor amplifier operates in the kHz range. This is far above the frequency range of interest in series elastic actuators and probably does not significantly affect the overall bandwidth performance. Hydraulic servo valves however have low frequency first order dynamics that strongly affect actuator bandwidth performance.

Sensor resolution can take on many forms: encoder ticks, noise floor of a potentiometer, hysteresis in strain gauges and discrete motor poles. Each of these reduce information passed either in the forward or feedback loops. The sensitivity and bandwidth of an actuator cannot be raised above the limitation set by the sensors. Noise in the sensors are amplified with increase controller gain which can lead to instability.

Finally, various items in the controller dynamics can affect the closed-loop bandwidth. Low-pass filtering a noisy sensor adds poles to the controller dynamics. A similar effect comes from using an integral term in a controller which adds phase lag. However, it may be necessary to filter signals or have PI control to get zero steady state error. Therefore, it is important to remember that the controller itself can be the limit in bandwidth.

There are various things that limit the actuator bandwidth. Depending on the design and configuration one of the limitation will be dominant. Increasing closed-loop bandwidth is an optimization problem between all of the potential limitations. It is interesting to point out, however, that the spring stiffness, $k_s$, in the sensor is *not* one of the limits. The stiffness of the spring does have implications to the saturation bandwidth of the actuator discussed later in section 3.2.5. Nevertheless, $k_s$ may be a high stiffness or very compliant, but the fundamental bandwidth limit is independent of $k_s$.

As an aid to understanding how the bandwidth is independent of $k_s$, notice that the spring constant $k_s$ and the control gain $K$ are always together in the loop gain. The key point of series elastic actuators is that the spring is very compliant in comparison to the stiffness of a load cell. Therefore, by decreasing $k_s$ it is possible to increase the control gain a proportional amount and maintain the same performance of the actuator. Series elastic actuators take the gain out of the sensor and put it into the control system.

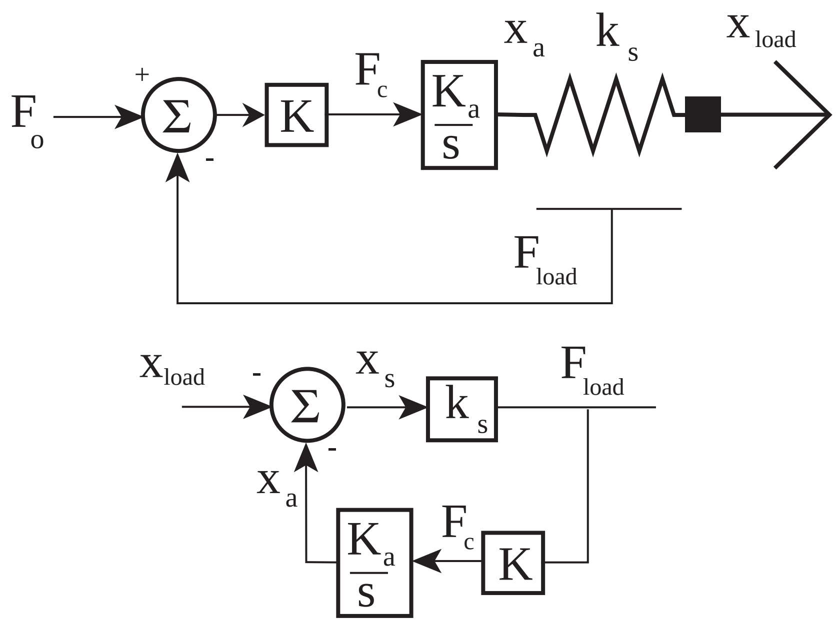


Figure 3-4: Free Load Model and Block Diagram. The impedance of the actuator is seen with load $x_l$ free to move.

### 3.2.2 Case 2: Free Load – Output Impedance

The second case of interest is when the load is free to move and the desired force is fixed constant $F_d = F_o$ (figure 3-4). With $F_d$ fixed, the dynamics due to the desired force are eliminated. Therefore, the transfer function relating the change in load force $F_l$ due to the load position $x_l$ can be written as

$$Z(s) = \frac{F_l}{x_l} = \frac{-k_s s}{s + k_s K K_a}. \tag{3.13}$$

This equation is referred to as the output impedance[1]. As described in the thesis introduction, mechanical impedance is adapted from the idea of impedance in electrical circuits [25]. Output impedance is a measure of the stiffness of a system. For example, the impedance of a spring is its spring constant because it relates an input position to output force. For robot actuators, low impedance is desirable.

The output impedance in equation 3.13 is minimal at low frequency. At high frequency, the impedance is equal to the spring stiffness of the physical spring in the sensor. Having minimal impedance is like having a zero rate spring connected to the load. Any load mass will move around and react to the actuator force commands almost entirely decoupled from the dynamics of the actuator itself.

There are two factors that reduce the impedance of the series elastic actuator, one passive and one active. The first is the fact that the spring stiffness $k_s$ is greatly reduced from that of a stiff load cell. This is a passive effect and the lower spring stiffness may reduce the impedance by several orders of magnitude in and of itself. Table 3.1 shows the difference in stiffness from an off-the-shelf load cell verses the springs used in the series elastic actuator prototypes. The difference is greater than two orders of magnitude.

The second effect in reducing output impedance comes from the active control system. Note that the characteristic equation for both closed-loop forward transfer function (equation 3.11) and

[1] $F_l$ has been adjusted to be centered on $F_o$. Therefore, this is the impedance around the constant desired force $F_o$

Table 3.1: Comparison of sensor stiffness.

| Sensor | Stiffness (lbs/in) |
|---|---|
| Transducer Techniques LBO-500 | $500,000$ |
| Century Die Spring D-1222-A | $4 \times 450 = 1,800$ |

closed-loop output impedance (equation 3.13) is the same.

$$s + k_s K K_a = 0 \tag{3.14}$$

The argument previously given in case 1 for increasing the control gain $K$ proportional to the reduction of spring stiffness $k_s$ holds for the impedance as well. Therefore, not only is there a passive impedance reduction with the compliant spring but the control system has an increased gain which further extends the low impedance range.

The output impedance has an active physical equivalent at low frequencies. Given that the load motion is slow enough or has low frequency content then the impedance looks like a physical damper.

$$F_l = b_{eq} s x_l \tag{3.15}$$

Where the equivalent damping coefficient is:

$$b_{eq} = \frac{1}{K K_a}. \tag{3.16}$$

The damping effect is strongest at low frequencies and the passive impedance of the spring dominates at higher frequencies. Depending on the actuator dynamics and the control system, the impedance takes on different physical equivalents at low frequency and is always equal to the spring at high frequencies. For this case, the closed-loop bandwidth of the system, $\omega_c$, is the frequency at which the damping effect and the passive impedance of the spring are the same. Regardless of the impedance bode profile, the increased compliance in the sensor dramatically reduces the overall output impedance of the actuator.

### 3.2.3 Error Rejection

In any physical system, there are non-linearities that are often neglected in a mathematical model. For example, in a transmission such as a ball screw or a gear train, there is stiction, coulomb friction and backlash. Although not physically identical, similar effects are found in hydraulics. There is stiction in the seals of the piston. There is also delay when switching between pressurized sides. Stiction and coulomb friction represent force errors in the transmission and backlash represent motion errors. Neither type of error is desirable in an actuator. Nevertheless, they are present and reducing the effects of the errors is beneficial to the actuator.

Since there is a large distance to force ratio on the sensor because of the compliant spring, the control system can have large gain and can respond very quickly to non-linearities such as backlash and friction. Due to the unique tradeoff of removing gain from the sensor to be in the control system, series elastic actuators have excellent internal error rejection capability. The following analysis outlines the error rejection for both internal force errors and position errors.

#### Force Error Rejection

Stiction and Coulomb friction are internal force errors that are reduced by using a compliant sensor and increased control gain. In the case of stiction, the actuator will not move from a static point until the static friction threshold force has been reached. Coulomb friction refers to the friction force present in a system which is independent of actuator velocity magnitude. I generalize each of these frictions depending on the state of the system and represent it as a non-linear friction force $F_f$.

At the point on the actuator side of the spring, the actuator will not move, in the case of stiction until

$$F_c - F_l \geq F_f \tag{3.17}$$

The same is true for coulomb friction.

Given that $F_c = KF_e$ from equation 3.7 we can conclude that the force error in the actuator due to stiction and coulomb friction is equal to:

$$F_e = \frac{F_f + F_l}{K} \tag{3.18}$$

Equation 3.18 shows that the effect of non-linear friction in a series elastic actuator is reduced in two ways: increased controller gain and increased load.

In any controlled physical system, increased controller gain has a direct reduction on disturbances. In this case, the controller can *break through* the stiction and minimize its effects. It is important to remember that since the spring sensor stiffness $k_s$ is reduced by several orders of magnitude, the gain $K$ can be increased by a proportional amount while maintaining stability. Therefore a series elastic actuator has dramatic force error rejection in comparison to an actuator with a stiff load cell because the controller gain can be set very high.

If the non-linear friction force $F_f$ is independent of the load force $F_l$, then for the simple proportional controller in the general model, the effect of $F_f$ is greatest when $F_l = 0$. The stiction or break away forces at this low force level are the limitation in low force sensitivity. This is the case for some non-linear frictions such as seal friction in a hydraulic piston. At higher load forces, $F_f$ becomes dominated by the increased load.

The other case where $F_f$ is a function of $F_l$ can also be true. In the simplest case $F_f$ is proportional to the load force such as in static or kinetic friction.

$$F_f = F(F_l) = \mu F_l \tag{3.19}$$

In the low force range, the residual stiction in the system still dominates actuator sensitivity. However, with increased load, the non-linear friction forces are still a source of error. Regardless, of the magnitude of the error, the increased controller gain makes the overall actuator system work better.

### Motion Error Rejection

In addition to force errors, there are motion errors in the actuator. A motion error means that for a given position or velocity at the motor there is some variation from that motion at the actuator output as expected in the linear model. An example of this is backlash in a gear train or ballscrew.

I represent a motion error as a distance $x_b$. A distance error corresponds to an equivalent force when seen through the spring sensor that must be compensated for by the controller.

$$F_c = x_b k_s \tag{3.20}$$

The force signal needed to correct this position error is defined by the general control law equation 3.7. Solving for the error, I get the error signal as a function of the position error, spring constant and controller gain.

$$F_e = \frac{x_b k_s}{K} \tag{3.21}$$

Equation 3.21 shows that the error in the system due to the the position error is a function of the ratio $k_s/K$. As discussed many times before, decreasing $k_s$ means that $K$ can be increased. Using a compliant sensor and increasing the control gain dramatically reduces the effect of motion errors like backlash in a series elastic actuator.

Assuming low output force, the force error due to internal non-linearity errors is effectively reduced by the inverse of the control gain, regardless of being force or motion based. This relationship can be seen in figure 3-5. The figure has been normalized to the magnitude of the internal error.

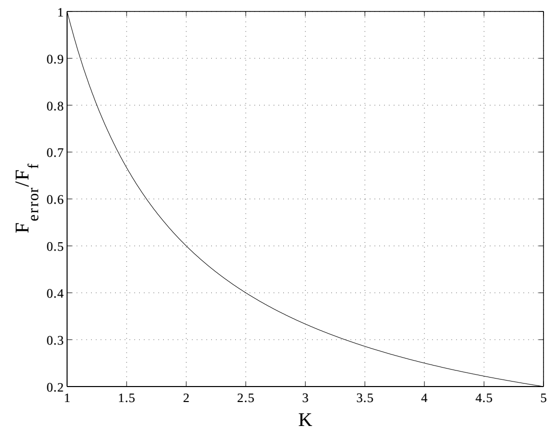


Figure 3-5: General Error Rejection. The magnitude of internal force and motion errors are reduced by the inverse of the controller gain. The figure has been normalized to the magnitude of the internal error.

### 3.2.4 Impact Tolerance

Whenever a robot works in unknown surroundings it makes and breaks contact with that environment. The interaction power at the system interface is subsequently transfered through the robot structure and can be quite significant. To minimize interaction power, typical force controlled robots move very slowly in anticipation of contact. If the impact power is too high, there is the potential for damage to the robot structure, actuators and to the environment itself. Series elastic actuators maximize impact tolerance by the use of the load bearing compliant sensor and can operate at high speeds even in the presence of potential impacts.

While the energy transfered to the actuator is the same regardless of spring stiffness, the spring and control system reduce the peak impact power to a range within the actuators output power capabilities. This power matching insures that the impact energy does not exceed the load capability of the actuator. The compliance allows the actuator to move out of the way of the impact while maintaining the proper desired force.

The impact power, $P_l$, from the load is defined by the force and velocity at the spring output.

$$P_l = F_l v_l \tag{3.22}$$

The spring compression defines $F_l$ but the environment defines $v_l$.

The general model for impedance (equation 3.13) can be redefined to be in terms of velocity rather than position.

$$\frac{F_l}{v_l} = \frac{k_s}{s + k_s K K_a} \tag{3.23}$$

Combining equations 3.22 and 3.23 gives the controlled impact power.

$$P_l = v_l^2 \frac{k_s}{s + k_s K K_a} \tag{3.24}$$

There are two parts to equation 3.24. The first is $v_l$ which is defined by the environment. There no control over this constraint. Therefore, in order to minimize interaction power, I focus on the second part of the equation which is due to the actuator.

The second part contains information about the actuator characteristics namely spring stiffness, control gain and bandwidth. Remember that this is the load interaction power regardless of the spring stiffness. The power contribution due to the actuator is at worst $1/KK_a$. As mentioned

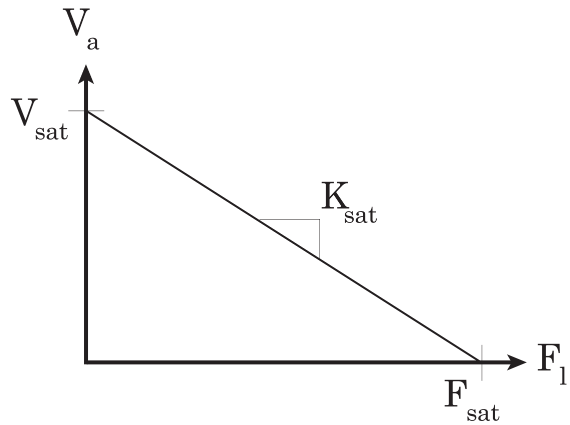


Figure 3-6: General saturation model. All actuator systems have limits to the maximum output capabilities. The actuator can operate anywhere within the bounded power region.

several times before, because of the sensor's compliance decreasing loop gain, the control gain $K$ can be increased. Clearly from equation 3.24, the higher the control gain, the lower the impact power. The impact power spectrum rolls off at higher frequency with the break point at the controlled bandwidth of the actuator but this effect is secondary.

### 3.2.5 Saturation Model

Up to this point there has been the assumption that the actuator was not operating in a saturated condition. It assumed that any command given by the controller could be achieved by the actuator system. A command input yielded a desired velocity output.

Power saturation of an actuator plays an important role in defining the the actuator's capabilities. Saturation plays an even more critical role when a significantly compliant element is placed in series between the power producing element of the actuator and the output. While the overall bandwidth of the actuator is not affected by the spring, the large force bandwidth is significantly altered. Large force bandwidth is defined as the frequency range over which the actuator can achieve a force equivalent to the maximum force $F_{sat}$ at steady state.

Because of the compliance, the actuator can only modulate the maximum force over a certain frequency range. This saturation frequency, hereafter referred to as $\omega_o$, is less than the bandwidth of the complete closed-loop system when using a very compliant sensor. The actuator can still produce '1force above $\omega_o$ but maximum force capability becomes less and less with increased frequency.

This can be understood intuitively by remembering that the power producing element in the actuator must move a significant amount in order to compress the compliant springs and generate the maximum force. If the actuator has velocity saturation then there is a limit to the frequency at which the actuator can oscillate at maximum force. Decreasing the stiffness of the spring means that the distance the actuator must move to achieve maximum force is increased. Thereby, decreasing the large force bandwidth. The opposite is true for increasing the stiffness.

I introduce a simple maximum force model with velocity saturation. Figure 3-6 shows that the maximum velocity (speed of response) is limited by the force present in the spring and the maximum force output capability of the actuator $F_{sat}$.

$$
\begin{aligned}
v_a = x_a s &= K_{sat}(F_{sat} - F_l) \\
\Rightarrow x_a &= \frac{K_{sat}}{s}(F_{sat} - F_l)
\end{aligned}
\tag{3.25}
$$

$K_{sat}$ is equal to $\frac{V_{sat}}{F_{sat}}$. It may be the case, that $K_{sat} = K_a$ but not necessarily.

This model and the following analysis is only concerned with the saturated condition when the actuator is working at the maximum output power capacity. Therefore, while the actuator can operate anywhere within the power envelope, in the saturation analysis the actuator only operates along the saturation line defined in figure 3-6.

To establish a baseline understanding of saturation effects in series elastic actuators, I neglect the load motion ($x_l = 0$). Combining equation 3.25 and the equation for the force in the spring shown in section 3.1.5, I obtain

$$\begin{aligned} F_l &= k_s(x_a - \cancel{x_o}) \\ &= k_s \frac{K_{sat}}{s}(F_{sat} - F_l). \end{aligned} \tag{3.26}$$

Rearranging equation 3.26 and normalizing by the maximum force $F_{sat}$, shows how well the actuator can achieve maximum force as a function of frequency.

$$\frac{F_{l_{max}}}{F_{sat}} = \frac{k_s K_{sat}}{s + k_s K_{sat}} \tag{3.27}$$

In this general case, the large force bandwidth or saturation bandwidth follows a first order profile. The break point is at the saturation frequency discussed earlier. Here it is is defined explicitly as the saturation frequency

$$\omega_o = k_s K_{sat}. \tag{3.28}$$

$\omega_o$ is a function of two elements in the open-loop dynamics of the power domain, $K_{sat}$ and $k_s$.

$K_{sat}$ represents the saturation characteristics of the actuator. It defines how much power can be put into the spring of the force sensor. $k_s$ is the spring constant of the sensor. As described many times before, $k_s$ is very compliant in a series elastic actuator. Therefore, the large force bandwidth is significantly lower than it would be if a stiff load cell were used in the feedback loop.

For many robotic applications, especially force control situations, having a large saturation bandwidth is unnecessary. The maximum output force of the actuators are only required at low frequencies. Having a reduced saturation bandwidth is the key engineering trade off for performance gains in the dramatically improved output impedance, error rejection and impact tolerance. It makes better sense to match the stiffness of the sensor to the requirements of a task. Therefore, choosing a proper $\omega_o$ for the actuator is important for making closed-loop actuator force control work properly.

Equivalent saturation frequencies can be defined for the large force bandwidth as a function of the open loop dynamics of the power domain for both electro-mechanical and hydraulic systems. This will be discussed in the following chapters.

## 3.3 Dimensional Analysis

Dimensional analysis often helps in understanding the basic relationships of all the different variables in an equation. The equations in this chapter are simple enough to understand without going through the formality of dimensionless group reformulation. However, rewriting the closed-loop forward transfer function and the output impedance from this chapter is a good exercise and will be helpful when applying dimensional analysis to the models in the case studies.

For the simple general model there are two dimensionless groups. The saturation frequency, $\omega_o$, defined in section 3.2.5 is used to normalize frequency. The other group is a generalized gain, $\kappa$.

$$\begin{aligned} S &= \frac{s}{\omega_o} = \frac{s}{k_s K_{sat}} \\ \kappa &= \frac{k_s K K_a}{\omega_o} \end{aligned} \tag{3.29}$$

- $S$ is a scaled Laplace variable which normalizes s to $\omega_o$.

- $\kappa$ is the normalized gain in the system. In this case it represents the increase in frequency of the controlled system above the saturation frequency.

#### Closed-loop Forward Dynamics

The closed-loop forward transfer function relating the output force as a function of the desired force is written in equation 3.11. It is rewritten here for convenience.

$$G(s) = \frac{F_l}{F_d} = \frac{k_s K K_a}{s + k_s K K_a}.$$

Applying the dimensionless groups from equation 3.29 gives

$$G(S) = \frac{F_l(S)}{F_d(S)} = \frac{\kappa}{S + \kappa}. \tag{3.30}$$

This equation shows that the DC gain of the closed-loop system is still unity. $\kappa$ indicates the breakoff frequency above the saturation frequency where the controlled system rolls off.

#### Closed-loop Output Impedance

The closed-loop output impedance is taken from equation 3.13. It is rewritten here.

$$G(s) = \frac{F_l}{x_l} = \frac{k_s s}{s + k_s K K_a}.$$

Applying the dimensionless groups from equation 3.29 gives

$$G(S) = \frac{F_l(S)}{k_s X_l(S)} = \frac{S}{S + \kappa}. \tag{3.31}$$

This equation is normalized to $k_s$ and shows that the impedance is very low and increases at 20dB/dec up to the controlled bandwidth of the actuator. Remember that this is an equivalent damper with a low damping coefficient. At the point of transition, the impedance is equal the spring constant of the physical spring.

Both the normalized closed-loop forward dynamics and the output impedance are shown graphically in figure 3-7.

#### Saturation Bandwidth

Similarly, I normalize the saturation bandwidth from equation 3.27.

$$\frac{F_l(S)}{F_{sat}(S)} = \frac{1}{S + 1} \tag{3.32}$$

This equation is pictured in figure 3-8. The saturation frequency is independent of the control system and is defined completely by the open-loop actuator dynamics and the spring constant $k_s$. Reduced saturation bandwidth is the tradeoff that must be made in the design of series elastic actuators.

## 3.4 General Model Summary

The general model helps to demonstrate some of the key ideas of using a significantly compliant force sensor for closed-loop actuator force control. In summary, the simple linear actuator model demonstrates and quantifies closed-loop bandwidth, output impedance, error rejection, and impact tolerance.

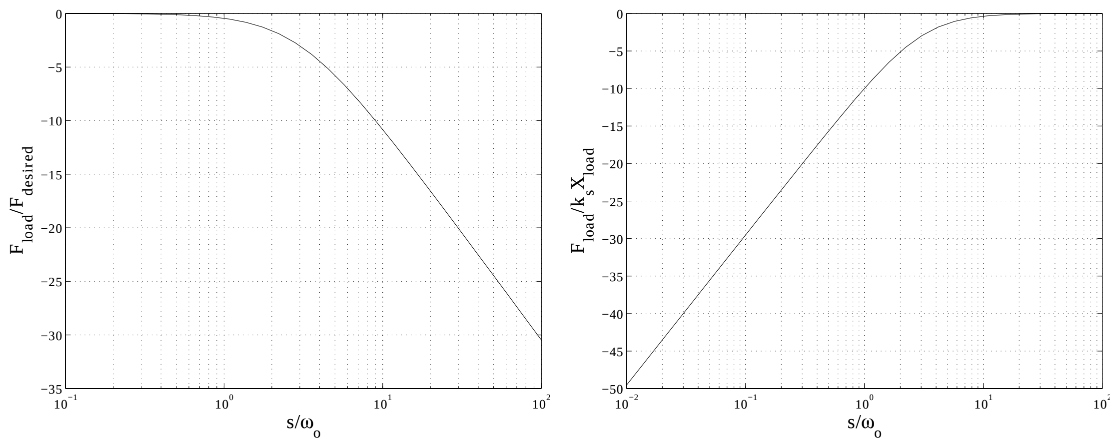


Figure 3-7: General closed-loop forward transfer function (left) and output impedance (right). These figures are normalized to the saturation frequency $\omega_o$ and have $\kappa = 3$.

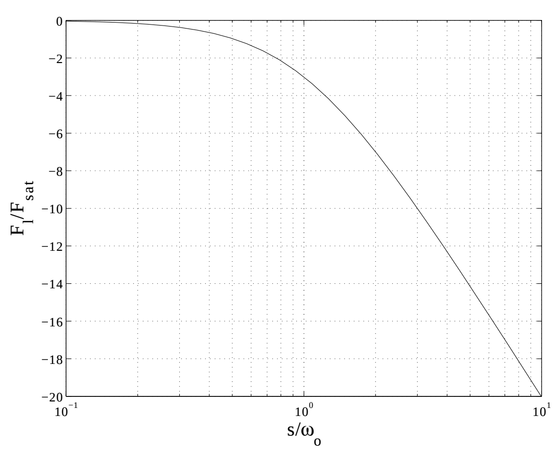


Figure 3-8: General large force saturation bandwidth. The ability of an actuator to output its full steady state force level is compromised by the introduction of a very compliant spring. $\omega_o$ is the break frequency and is defined by the open loop dynamics of the actuator. The large force bandwidth is typically less than the controlled bandwidth of the actuator. Even though the actuator cannot achieve full force output at high frequency, it can and does operate above $\omega_o$ .

- **Closed-loop bandwidth** is *independent* of the spring constant. The reduction in sensor gain can be compensated for an increase in controller gain. The bandwidth is limited by other factors such as non-colocated dynamics and valve and amplifier limitations.

- Low **output impedance** in the actuator is achieved by decreased spring stiffness and increased controller gain. Even stiff flow source actuator systems can be made to be actively backdriveable and have an impedance no greater than the spring stiffness $k_s$ of the sensor.

- **Error reduction** of internal force and position error is achieved by decreasing sensor stiffness and increased controller gain. The effects of many non-linearities such as transmission backlash and stiction are minimized.

- **Impact tolerance** comes from the spring absorbing energy from the environment which allows the actuator control system to minimize peak impact power. This protects actuator components from overload damage.

The above characteristics of the actuator make series elastic actuators good force output devices. The gains in improvement come with one tradeoff.

- Large force bandwidth (saturation bandwidth) is significantly reduced with the compliant sensor. The actuator can still operate above the saturation bandwidth even though the maximum force output capability is limited. Large force bandwidth is independent of control system.

The above ideas are talked about and described in further detail as they apply to the specific case studies of applying series elasticity to the electro-mechanical domain and hydraulic domain.

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