6.824 Artificial Intelligence, Fall 1994
Problem Set 6 Solutions

Problem 1. Assume the following placement of two queens on the chessboard:

	1	2	3	4	5	6
1			X			
2						
3	O		X			
4			X			
5		O	X			
6			X			

In the above example GFC derives the equation $Q_3 = 2$ from disequations by exhaustion of alternatives.

Value propagation fails to derive that equation because value propagation doe no tderive disequations and does not derive values by exhaustion.

Problem 2. Assume the partial placement of queens on the 6x6 chess board.

GFC

	1	2	3	4	5	6
1	O		X	X	X	X
2			X			
3		O	X	X	X	X
4			X	X		
5				X	X	
6					X	X

Arc Consistency

	1	2	3	4	5	6
1	O		X	X	X	X
2			X			
3		O	X	X	X	X
4			X	X		
5				X	X	
6				**X**	X	X

GFC cannot derive any equation for this partial placement. This is because:

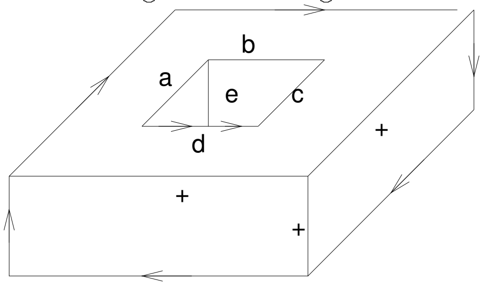
1. for every unoccupied column there are at least 2 positions for which disequations were not derived by GFC (disequations can be derived only from equations), and
2. GFC can derive an equation only from equations or from disequations by exhaustion of alternatives

Arc consistency can derive equation $Q_4 = 2$, because:

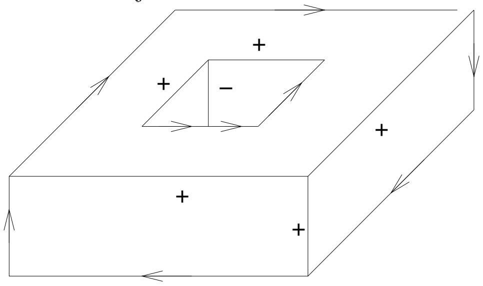
1. There are 2 possibilities for placing Q_3: $Q_3 = 5$ or $Q_3 = 6$. Both of them prevent Q_4 to be placed in row 6. Therefore disequation $Q_4 \neq 6$ can be derived.

2. Using all derived disequations for Q_4 the equation $Q(4) = 2$ can be derived by exhaustion of alternatives.

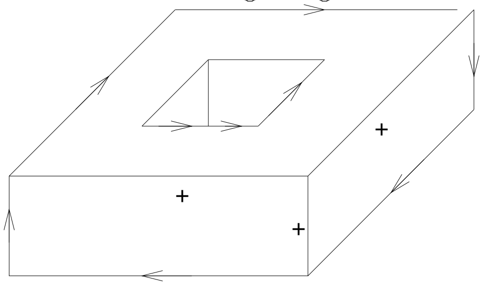
Problem 3. Consider the following line drawing



Starting with the arrows around the border, there is a unique assignment to the lines in this drawing which is entailed by the constraints. This assignment is



Value propagation will find the following assignment



which leaves unlabeled some edges in the hole. Line c can be counter-clockwise arrow or +, and line a can be counter-clockwise arrow or +. The arrow junction at aeb is unconstrained and cannot be assigned. So value propagation is stuck at this point and therefore it is incomplete.

Note that *Arc consistency propagation* can find the complete assignment to the hole lines as follows. From the arrow junction at aeb it finds $a \neq \leftarrow$ (not a counterclockwise arrow). Thus $a = +$, and from this the arrow junction is labeled with $b = +$ and $e = -$. Now the L junction at bc constrains $c = \leftarrow$ (counter-clockwise arrow).

Problem 4. Show that arc consistency is complete for Horn clauses.
Remember that a *constraint propagation algorithm* is called complete if it can derive all assignments that are entailed by the constraints.

Consider an unsatisfiable set of Horn clauses. We must show that if there is no assignment satisfying these Horn clauses then the arc consistency propagation procedure will return "inconsistent". We will prove this by showing that the contra-positive is true. *If arc consistency propagation fails to derive an inconsistency then the given set of Horn clauses is satisfiable.*

Suppose we have some set of Horn clauses Σ and that unit propagation does not find an inconsistency and outputs a partial assignment. We can divide the clauses into two sets: (1) clauses that are satisfied by the partial assignment, and (2) clauses that are not satisfied by the partial assignment.

Consider the clauses in set (2). Each of these clauses has at least 2 literals which are not bound in the partial assignments, because if only one literal remained unbound and the clause was not yet satisfied that literal would be assigned by unit propagation. Furthermore, these clauses contain at least one unassigned literal which is the negation of a variable, because each clause has at most one positive literal. So we can satisfy all the clauses by assigning all the unbound variables to false. Thus Σ is satisfiable.