6.824 Artificial Intelligence, Fall 1994
Problem Set 8

Due at class Wednesday, Nov. 30

Problem 1. This problem is on nonlinear planning. The Sussman anomaly is a blocks world planning problem where the initial state is given by the propositions

`CLEAR(C),   ON(C,A),  ON(A,PLACE1)`

`CLEAR(B),  ON(B,PLACE2),  CLEAR(PLACE3)`

and the goal consists of the two propositions `ON(A,B)`, `ON(B,C)`. Consider applying the non-linear planning procedure in the notes to the Sussman anomaly. Draw a complete nonlinear plan, i.e., one with no threats or open preconditions, that is a solution to the Sussman anomaly and that can be produced by the nonlinear planner in the notes. You should show all the causal links and safety conditions. In addition to the drawing, provide a list of the safety conditions and for each safety condition give a causal link that would be threatened if that safety condition were not present.

Problem 2. This problem is on lifting. Consider rewrite rules such as the following.

$$(*\ x\ (+\ y\ z)) \rightarrow (+\ (*\ x\ y)\ (*\ x\ z))$$

$$(+\ (+\ x\ y)\ z) \rightarrow (+\ x\ (+\ y\ z))$$

$$x \rightarrow (+\ x\ (+\ y\ (-\ y)))$$

Each rewrite rule specifies that an instance of the left hand side can be rewritten as an instance of the right hand side. For example, the first rule specifies that $(*\ (f\ a)\ (+\ c\ d))$ can be rewritten to $(+\ (*\ (f\ a)\ c)\ (*\ (f\ a)\ d))$. The last rule specifies that $(f\ a)$ can be rewritten to $(+\ (f\ a)\ (+\ (g\ b)\ (-\ (g\ b))))$.

The above rules are only meant as an example — in this problem we consider the general concept of rewrite rules. Let R be a set of rewrite rules and let s and w be ground expressions (expressions that do not contain variables). Suppose that we use the following procedure to determine if there exists a way of rewriting s to w using rewrite rules in R using at most d rewriting steps.

Procedure to determine if s can be rewritten to w in d steps

1. If s is the same expression as w then return "yes".
2. If d is 0 then fail.
3. Select (nondeterministically) be a *ground instance* $u \rightarrow v$ of a rewrite rule in R.

4. If u does not equal s then fail.

5. If u equals s then return the value of a recursive call to the procedure on v, w and $d-1$.

The above procedure operates on ground expressions (step 3 selects a ground instance of a rewrite rule). The general lifting transformation can be applied to this procedure. The following questions concern the differences between the ground procedure and the lifted procedure.

Assume that at least one rule in the rule set R contains a variable and that there are n different rules in R. In the ground procedure given above, how many ways are there of executing step 3? In the lifted version of the procedure, what is the maximum number of ways that the sequence of step 3 followed by step 4 can execute without failure? Consider a call to the lifted version of the procedure. As a function of n (the number of rules) and d (the allowed number of steps), give the maximum possible number of leaves in the search tree, i.e., the maximum number of times some recursive call to the procedure can fail at step 2.

Problem 3. Consider a minmax game tree in which each max node has a branching factor of $b_{\max}$ and each min node has a branching factor of $b_{\min}$. Now consider an α-β search in which the best move is tried first at each ply and so that an optimal α-β search tree is generated. In a d-ply tree starting at a max node how many leaf nodes will be examined? Your answer should be given in terms of the parameters d, $b_{\max}$, and $b_{\min}$.

Problem 4. Consider the minmax game tree described in problem 1. However, rather than a perfect α-β, consider a "full" d-ply tree, i.e., one which contains all nodes d-ply or less from the root. Furthermore, suppose that every leaf node in this tree has the same static value. What is the conspiracy depth of a leaf node in this tree with resect to an target value larger than the root value? What is the conspiracy depth of a leaf node in this tree with respect to a target value smaller than the root value? Both answers should be given in terms of d, $b_{\max}$ and $b_{\min}$.