Lecture 8
Self-calibration of a moving camera

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Introduction

- We want to eliminate the need for the calibration pattern discussed in lecture 4.
- We will use the fact that when the camera moves while keeping its internal parameters fixed, the image of the absolute conic Ω does not change.
- Ω, through the estimated F-matrix or through the estimated infinity collineation will effectively be used as a calibration pattern!

Constraints on the Kruppa matrices

- Assume that we have estimated $\mathbf{H}_\infty$: pure translation or rotation through an axis going through the optical center.
- We have the following proposition:

Proposition 1 *Given the infinity collineation $\mathbf{H}_\infty$ between two views, the relation between the Kruppa matrices is the following:*

$$\mathbf{K}' = \mathbf{H}_\infty \mathbf{K} \mathbf{H}_\infty^T$$

- This places five constraints on the intrinsic parameters of the two views.
- A natural question to ask is whether these constraints are sufficient to recover the intrinsic parameters of the camera in the case where they are the same in the two views.

Constraints on the Kruppa matrices (*continued*)

- Unfortunately, the answer to this question is no as shown in the next

Proposition 2 *The solutions to the matrix equation:*

$$\mathbf{H}_\infty \mathbf{K} \mathbf{H}_\infty^T \simeq \mathbf{K}$$

where $\mathbf{H}_\infty$ *is known and the symmetric matrix* $\mathbf{K}$ *is unknown are given by:*

$$\mathbf{K} = \lambda \mathbf{A}\mathbf{A}^T + \mu \mathbf{A}\mathbf{r}\mathbf{r}^T\mathbf{A}^T$$

where λ *and* μ *are arbitrary constants and* $\mathbf{r}$ *is any vector satisfying* $\mathbf{R}\mathbf{r} = \mathbf{r}$

Constraints on the Kruppa matrices (*continued*)

- We can hope that if we perform two motions of the camera such that the corresponding rotation matrices are "sufficiently different", we should be able to recover the intrinsic parameters.
- Indeed, we have the following theorem:

Theorem 1 *Let* $\mathbf{H}_{\infty i}, i = 1, 2$ *be two infinity collineations and* $\mathbf{K}$ *be an unknown symmetric matrix. The solutions* $(x_1, x_2, \mathbf{K})$ *of the system of two equations:*

$$\mathbf{H}_{\infty i}\mathbf{K}\mathbf{H}_{\infty i}^T - x_i^2\mathbf{K} = \mathbf{0} \;\; i = 1, 2$$

are given by:

$$x_i \in \{-1, 1\} \;\; i = 1, 2 \;\; \textit{and}$$

$$\mathbf{K} \in \begin{cases} \langle \mathbf{A}\mathbf{A}^T \rangle & \textit{if the two rotations have different axes} \\ \langle \mathbf{A}\mathbf{A}^T, \, \mathbf{A}\mathbf{r}_1\mathbf{r}_1^T\mathbf{A}^T \rangle & \textit{otherwise} \end{cases}$$

One constraint on the internal parameters

- Consider two cameras with optical centers C and C', internal parameters $\mathbf{A}$ and $\mathbf{A}'$.
- We estimate the F-matrix between the two views. It is related to the E-matrix by:

$$\mathbf{E} = \mathbf{A}'^T \mathbf{F} \mathbf{A}$$

- The Huang and Faugeras equations:

$$\det(\mathbf{E}) = 0 \qquad f(\mathbf{E}) = \frac{1}{2}\mathrm{trace}^2(\mathbf{E}\mathbf{E}^T) - \mathrm{trace}(\mathbf{E}\mathbf{E}^T)^2,$$

- impose one algebraic constraint of degree 8 on the internal parameters.

The Kruppa equations

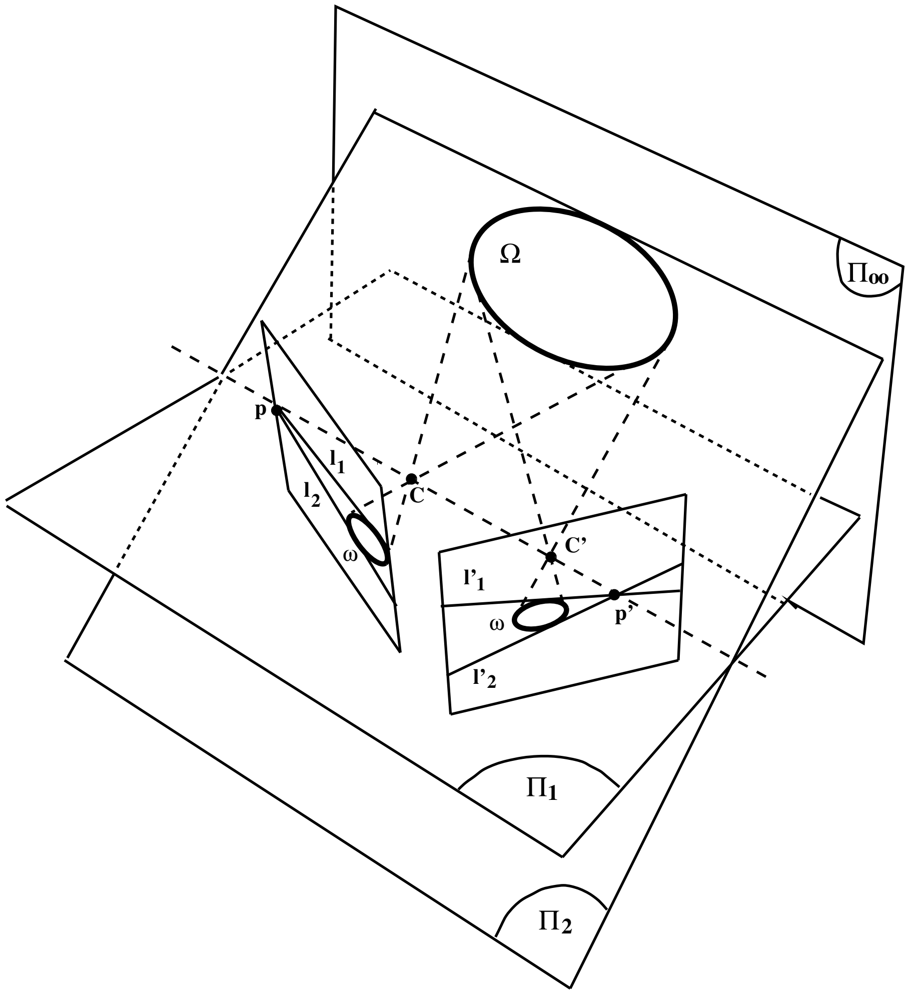


Figure 1: The absolute conic and epipolar transformation.

The Kruppa equations (*continued*)

- Let us consider an epipolar plane Π, which is tangent to Ω.
- The corresponding epipolar lines l and l' are tangent to the images ω and ω' of Ω, respectively.
- It follows that the two tangents drawn to ω from the epipole $\mathbf{e}$ are in correspondence under the epipolar transformation with the two tangents drawn to ω' from the epipole $\mathbf{e}'$.

The Kruppa equations (*continued*)

- This has the following effect on the Kruppa matrices:

Proposition 3 *The two Kruppa matrices satisfy the following relation:*

$$[\mathbf{e}]_\times \mathbf{K} [\mathbf{e}]_\times \simeq \mathbf{F}^T \mathbf{K}' \mathbf{F}$$

which is equivalent to two polynomial equations of degree 2 in the coefficients of $\mathbf{K}$ *and* $\mathbf{K}'$*. These equations are called the Kruppa equations.*

- The nice thing with the Kruppa equations is that they are only of degree *two* in these entries, and therefore of degree 4 in the entries of $\mathbf{A}$ and $\mathbf{A}'$.

- Furthermore, we have the following equivalence:

Proposition 4 *The two Kruppa equations are equivalent to the constraint obtained from the second Huang-Faugeras equations.*

Degenerate cases

- The Kruppa equations are not defined in the case of a null translation. In that case we can use the method described above.
- The Kruppa equations are not defined in the case of a pure translation: $\mathbf{F} \simeq [\mathbf{e}]_{\times}$.
- The Kruppa equations are degenerate if the axes of rotation are the same.

Minimum number of views

- Since one displacement yields two Kruppa equations, the minimum number of views to solve for the five intrinsic parameters is three.
- In practice, the solution to the minimal case is used as an initial estimate for a larger minimization problem using more views.
- We estimate the three fundamental matrices and write the six Kruppa equations.
- We obtain five systems of polynomial equations of degree two in five unknowns which we solve by homotopic methods.
- Each system can produce as many as 2^5 solutions!
- Other methods make "reasonable" assumptions on the intrinsic parameters, such as $\theta = \frac{\pi}{2}$ or c_p approximately in the center of the image.

Practical solution

- We consider $N \geq 3$ estimated fundamental matrices $\mathbf{F}_i$ corresponding to a number of views taken with a camera with constant intrinsic parameters.
- Let $\mathbf{\Lambda}_{\mathbf{F}_i}$ be the corresponding 9×9 covariance matrices.
- Let $P_1(\mathbf{F}_i, \mathbf{K})$ and $P_2(\mathbf{F}_i, \mathbf{K})$ the two Kruppa equations for each pair of view.
- The first idea consists in minimizing:

$$C(\mathbf{K}) = \sum_{i=1}^{n} (P_1^2(\mathbf{F}_i, \mathbf{K}) + P_2^2(\mathbf{F}_i, \mathbf{K}))$$

  with respect to $\mathbf{K}$.
- The symmetric matrix $\mathbf{K} = \mathbf{A}\mathbf{A}^T$ is parameterized by the internal parameters $\alpha_u, \alpha_v, \theta, u_0, v_0$ or rather by $\alpha = \alpha_u, \beta = \frac{\alpha_v}{\sin\theta}, \gamma = \alpha_u \cot\theta, u_0, v_0$.

Practical solution (*continued*)

- This approach does not work in general because the uncertainty on the fundamental matrices is not taken into account.
- The correct approach is to minimize:

$$C(\mathbf{K}) = \sum_{i=1}^{n} \left(\frac{P_1^2(\mathbf{F}_i, \mathbf{K})}{\frac{\partial P_1}{\partial \mathbf{F}_i}(\mathbf{F}_i, \mathbf{K}) \mathbf{\Lambda}_{\mathbf{F}_i} \frac{\partial P_1}{\partial \mathbf{F}_i}^T (\mathbf{F}_i, \mathbf{K})} + \frac{P_2^2(\mathbf{F}_i, \mathbf{K})}{\frac{\partial P_2}{\partial \mathbf{F}_i}(\mathbf{F}_i, \mathbf{K}) \mathbf{\Lambda}_{\mathbf{F}_i} \frac{\partial P_2}{\partial \mathbf{F}_i}^T (\mathbf{F}_i, \mathbf{K})} \right)$$

- The minimization is then performed with respect to:
  1. 5 parameters.
  2. 4 parameters if $\theta = \frac{\pi}{2}$.
  3. 4 parameters if the aspect ratio $\delta_0 = \frac{\beta}{\alpha}$ is known.
  4. 3 parameters if we combine the two previous cases.
- The nonlinear minimization is initialized with the medians of the values of the analytical solutions.

Synthetic sequence 1

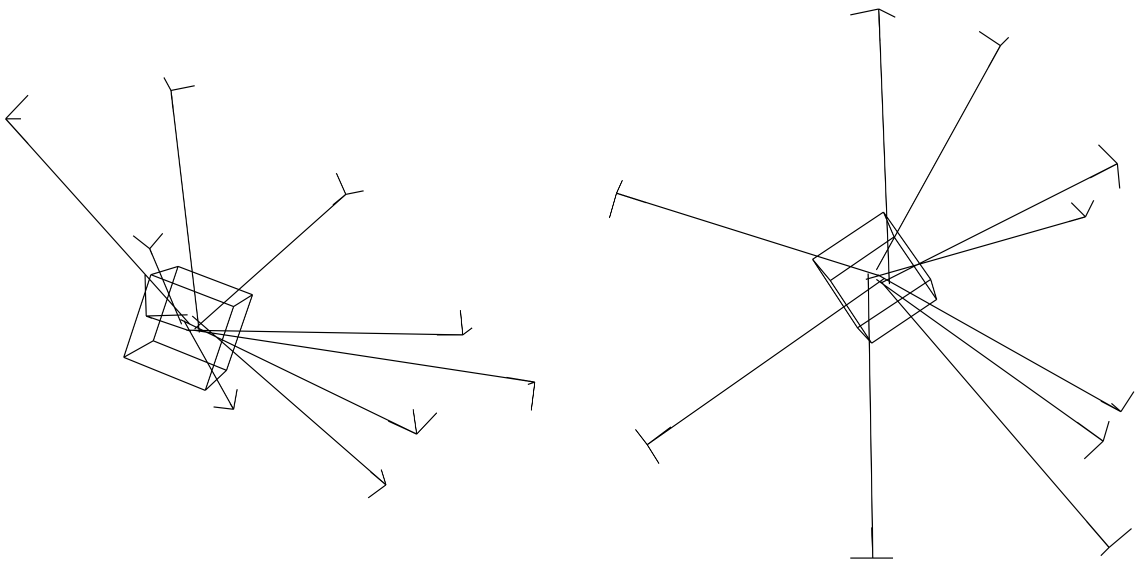
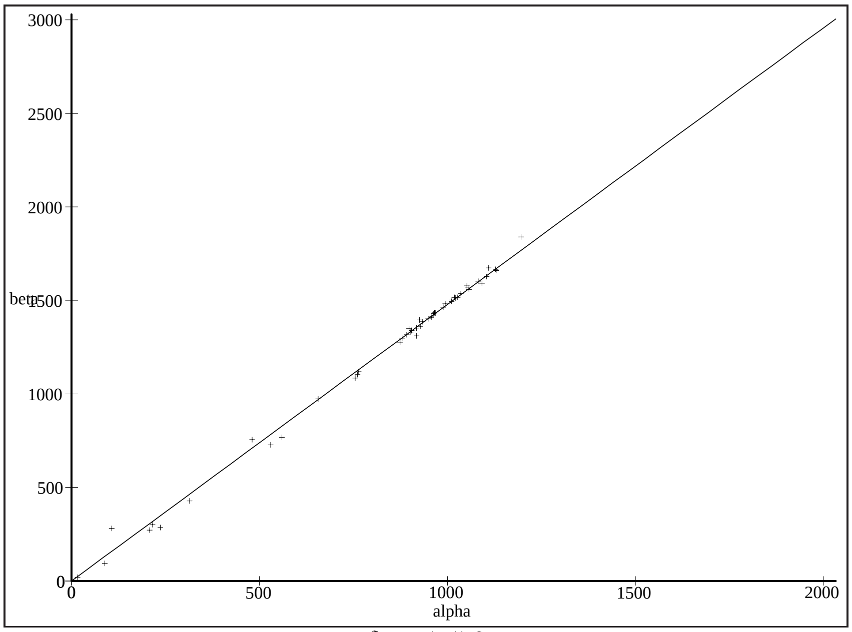


Figure 2: **Synthetic sequence 1** : 10 views; the observed scene lies within the cube.

Initialisation			

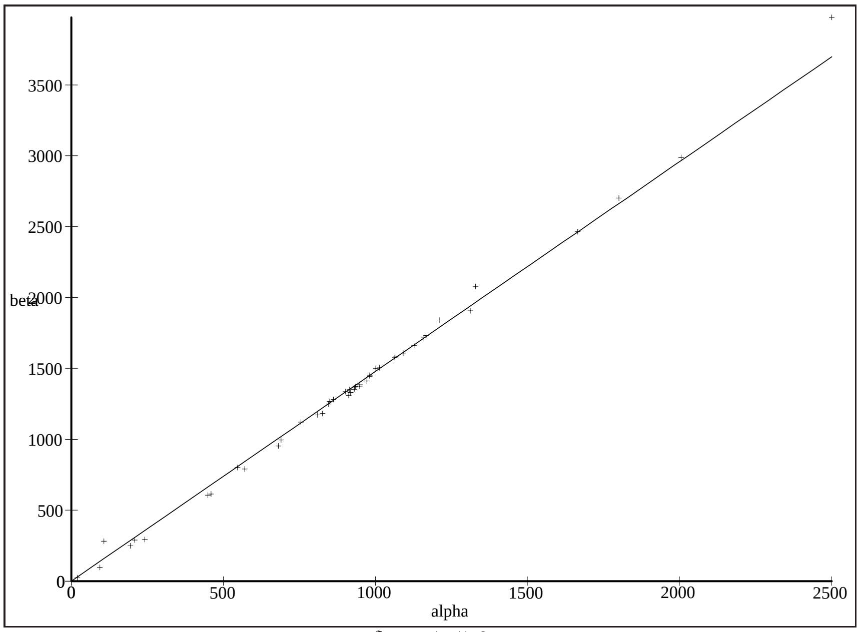


$\delta_0 = 1.478$

Median		Exact value	
$\mathbf{A} = \begin{bmatrix} 930.740 & 0 & 256 \\ 0 & 1390.937 & 256 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.494$	$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249.055 \\ 0 & 1352.105 & 258.933 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$
Minimisation with 5 parameters			
$\mathbf{A} = \begin{bmatrix} 913.848 & 0.036 & 248.939 \\ 0 & 1351.061 & 258.622 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 913.891 & 0.036 & 248.940 \\ 0 & 1351.124 & 258.621 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$
Minimisation with 4 parameters : $\gamma = 0$			
$\mathbf{A} = \begin{bmatrix} 913.941 & 0 & 248.949 \\ 0 & 1351.202 & 258.634 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 913.918 & 0 & 248.948 \\ 0 & 1351.168 & 258.635 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$
Minimisation with 4 parameters : $\beta = \delta_0 \alpha$			
$\mathbf{A} = \begin{bmatrix} 913.978 & 0.051 & 249.029 \\ 0 & 1351.012 & 258.557 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 913.978 & 0.051 & 249.029 \\ 0 & 1351.012 & 258.557 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$
Minimisation with 3 parameters : $\gamma = 0$ et $\beta = \delta_0 \alpha$			
$\mathbf{A} = \begin{bmatrix} 914.028 & 0 & 249.044 \\ 0 & 1351.085 & 258.557 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 914.028 & 0 & 249.044 \\ 0 & 1351.085 & 258.557 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$

Table 1: **Synthetic sequence 1** : intrinsic parameters (100 points - uniform noise, amplitude 0.1).

Initialisation



$\delta_0 = 1.479$

Median		Exact value	
$\mathbf{A} = \begin{bmatrix} 924.489 & 0 & 256 \\ 0 & 1351.365 & 256 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.462$	$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249.055 \\ 0 & 1352.105 & 258.933 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$
Minimisation with 5 parameters			
$\mathbf{A} = \begin{bmatrix} 934.179 & -0.312 & 252.999 \\ 0 & 1379.274 & 254.475 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.476$	$\mathbf{A} = \begin{bmatrix} 936.142 & -0.321 & 253.060 \\ 0 & 1382.183 & 254.388 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.476$
Minimisation with 4 parameters : $\gamma = 0$			
$\mathbf{A} = \begin{bmatrix} 933.605 & 0 & 252.918 \\ 0 & 1378.404 & 254.362 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.476$	$\mathbf{A} = \begin{bmatrix} 934.203 & 0 & 252.932 \\ 0 & 1379.291 & 254.332 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.476$
Minimisation with 4 parameters : $\beta = \delta_0 \alpha$			
$\mathbf{A} = \begin{bmatrix} 933.650 & -0.393 & 252.136 \\ 0 & 1380.508 & 254.942 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.479$	$\mathbf{A} = \begin{bmatrix} 933.603 & -0.396 & 252.090 \\ 0 & 1380.438 & 254.935 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.479$
Minimisation with 3 parameters : $\gamma = 0$ et $\beta = \delta_0 \alpha$			
$\mathbf{A} = \begin{bmatrix} 933.099 & 0 & 252.024 \\ 0 & 1379.692 & 254.942 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.479$	$\mathbf{A} = \begin{bmatrix} 933.038 & 0 & 251.979 \\ 0 & 1379.603 & 254.935 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.479$

Table 2: **Synthetic sequence 1** : intrinsic parameters (100 points - uniform noise amplitude 1.5).

	0.1		0.5		1		1.5	
	mean	std	mean	std	mean	std	mean	std
α	915.332	2.026	914.817	9.717	908.619	20.903	903.007	24.837
β	1353.190	2.995	1352.407	14.369	1343.144	30.860	1334.706	36.954
γ	-0.007	0.052	-0.014	0.289	-0.017	0.534	-0.089	0.919
u_0	249.068	0.235	249.168	1.327	248.898	2.542	248.645	3.779
v_0	258.885	0.268	258.755	1.202	259.160	2.150	259.511	3.716

Table 3: **Synthetic sequence 1** : means and standard deviations of the intrinsic parameter. The minimisation has been performed with respect to five parameters.

	0.1		0.5		1		1.5	
	mean	std	mean	std	mean	std	mean	std
α	915.340	2.026	914.835	9.690	908.584	20.806	903.080	24.974
β	1353.202	2.996	1352.433	14.336	1343.089	30.716	1334.805	37.182
u_0	249.067	0.235	249.162	1.327	248.903	2.532	248.642	3.743
v_0	258.882	0.268	258.746	1.166	259.154	2.123	259.485	3.628

Table 4: **Synthetic sequence 1** : means and standard deviations of the intrinsic parameter. The minimisation has been performed with respect to four parameters ($\theta = \pi/2$).

	0.1		0.5		1		1.5	
	mean	std	mean	std	mean	std	mean	std
α	914.680	1.736	914.848	9.361	909.766	17.246	910.702	22.748
β	1352.474	2.758	1352.246	14.390	1343.592	25.995	1344.262	36.163
γ	-0.014	0.071	-0.004	0.314	0.083	0.597	0.063	0.989
u_0	248.948	0.482	249.214	1.698	249.509	2.963	249.628	4.429
v_0	258.956	0.348	258.705	1.288	258.774	2.523	258.780	4.006

Table 5: **Synthetic sequence 1** : means and standard deviations of the intrinsic parameter. The minimisation has been performed with respect to four parameters (δ is fixed).

	0.1		0.5		1		1.5	
	mean	std	mean	std	mean	std	mean	std
α	914.661	1.738	914.817	9.371	909.815	17.270	910.567	22.813
β	1352.446	2.728	1352.200	14.382	1343.663	25.997	1344.061	36.164
u_0	248.944	0.494	249.210	1.719	249.519	3.009	249.630	4.479
v_0	258.956	0.348	258.705	1.288	258.774	2.523	258.780	4.006

Table 6: **Synthetic sequence 1** : means and standard deviations of the intrinsic parameter. The minimisation has been performed with respect to three parameters ($\theta = \pi/2$ and δ is fixed).

Noise	Computed with $\mathbf{\Lambda_F}$	Computed without $\mathbf{\Lambda_F}$
0.1	$\mathbf{A} = \begin{bmatrix} 914.028 & 0 & 249.044 \\ 0 & 1351.085 & 258.557 \\ 0 & 0 & 1 \end{bmatrix}$	$\mathbf{A} = \begin{bmatrix} 900.389 & 0 & 249.420 \\ 0 & 1330.925 & 260.407 \\ 0 & 0 & 1 \end{bmatrix}$
0.5	$\mathbf{A} = \begin{bmatrix} 881.131 & 0 & 246.681 \\ 0 & 1302.417 & 259.611 \\ 0 & 0 & 1 \end{bmatrix}$	$\mathbf{A} = \begin{bmatrix} 853.273 & 0 & 249.045 \\ 0 & 1261.239 & 257.493 \\ 0 & 0 & 1 \end{bmatrix}$
1	$\mathbf{A} = \begin{bmatrix} 905.662 & 0 & 246.157 \\ 0 & 1340.663 & 259.645 \\ 0 & 0 & 1 \end{bmatrix}$	$\mathbf{A} = \begin{bmatrix} 845.363 & 0 & 276.291 \\ 0 & 1251.402 & 201.631 \\ 0 & 0 & 1 \end{bmatrix}$
1.5	$\mathbf{A} = \begin{bmatrix} 933.099 & 0 & 252.024 \\ 0 & 1379.692 & 254.942 \\ 0 & 0 & 1 \end{bmatrix}$	$\mathbf{A} = \begin{bmatrix} 854.828 & 0 & 255.567 \\ 0 & 1263.960 & 246.710 \\ 0 & 0 & 1 \end{bmatrix}$

Table 7: **Synthetic sequence 1** : comparison between the results obtained with and without taking into account the uncertainty on the fundamental matrices.

	$\Delta\theta_{\mathbf{r}}$		$\Delta\theta$		$\Delta\theta_{\mathbf{t}}$	
Noise	mean	std	mean	std	mean	std
0.1	0.000613079	0.000599172	0.000556496	0.000442643	0.00103941	0.00115602
0.5	0.00231722	0.00219846	0.00226991	0.00162806	0.00397747	0.0042233
1.0	0.00469724	0.00440143	0.00466584	0.00451579	0.00637763	0.00771751
1.5	0.00704855	0.0072971	0.00698344	0.00665538	0.00839817	0.0103394

Table 8: **Synthetic sequence 1** : extrinsic parameters (9 displacements - $\Delta\theta_{\mathbf{r}}$, error on the direction of the axis of rotation (radian) - $\Delta\theta$, error on the angle of rotation (radian) - $\Delta\theta_{\mathbf{t}}$, error on the direction of translation (radian)).

Synthetic sequence 2

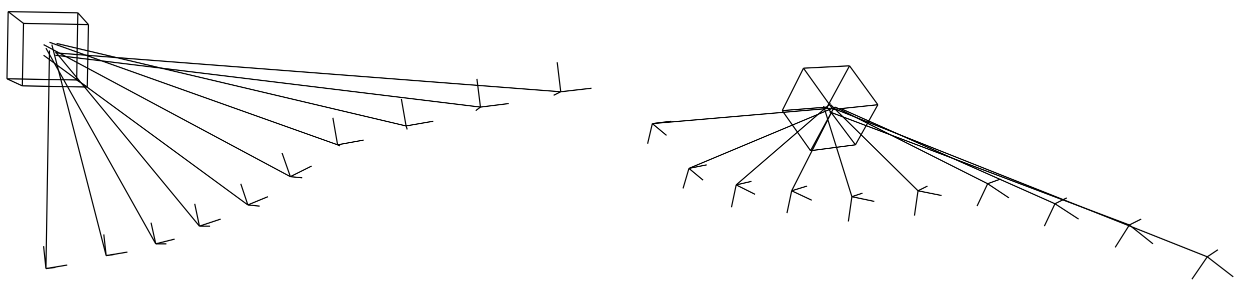


Figure 3: **Synthetic sequence 2** : 10 views for which the axes of rotation are parallel.

1st exact value		2nd exact value	
$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249 \\ 0 & 1352.105 & 258 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 902.892 & 5.655 & 257.728 \\ 0 & 1341.132 & 284.579 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.485$
Minimisation with 5 parameters			
$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249.055 \\ 0 & 1352.105 & 258.933 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 902.892 & 5.655 & 257.728 \\ 0 & 1341.132 & 284.579 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.485$
Minimisation with 4 parameters : $\gamma = 0$			
$\mathbf{A} = \begin{bmatrix} 914.572 & 0 & 249.068 \\ 0 & 1352.089 & 258.972 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$	$\mathbf{A} = \begin{bmatrix} 914.572 & 0 & 249.068 \\ 0 & 1352.089 & 258.972 \\ 0 & 0 & 1 \end{bmatrix}$	$\delta = 1.478$

Table 9: **Synthetic sequence 2** : There exists an infinity of possible intrinsic parameters, among which only one satisfies $\gamma = 0$.

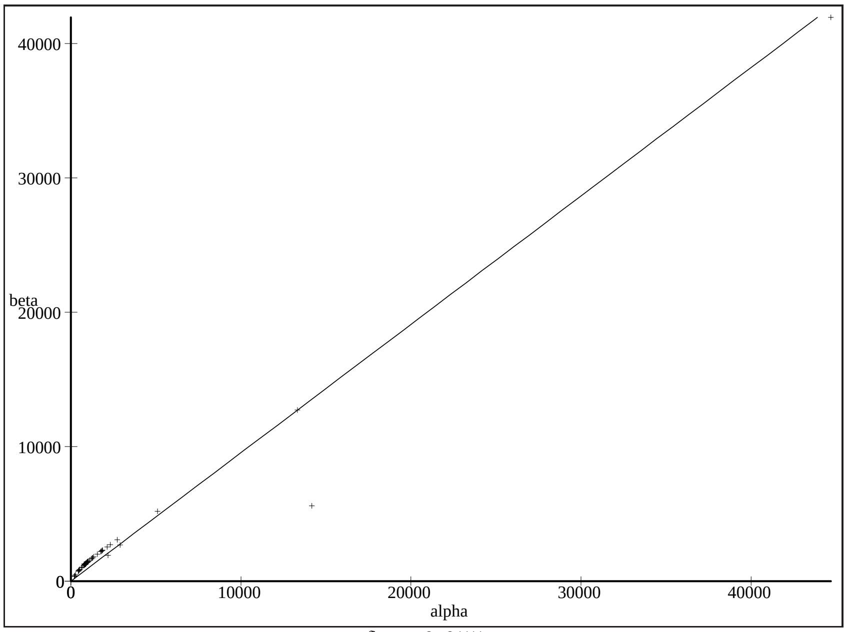
Initialisation	
	
$\delta_0 = 0.955$	
Median	**Exact value**
$\mathbf{A} = \begin{bmatrix} 913.550 & 0 & 256 \\ 0 & 1341.042 & 256 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.468$	$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249.055 \\ 0 & 1352.105 & 258.933 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.478$
Minimisation with 5 parameters	
$\mathbf{A} = \begin{bmatrix} 1110.838 & -200.330 & 52.161 \\ 0 & 1461.394 & -328.795 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.316$	$\mathbf{A} = \begin{bmatrix} 1111.083 & -201.120 & 51.705 \\ 0 & 1461.237 & -330.238 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.315$
Minimisation with 4 parameters : $\gamma = 0$	
$\mathbf{A} = \begin{bmatrix} 924.919 & 0 & 240.941 \\ 0 & 1369.242 & 238.532 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.480$	$\mathbf{A} = \begin{bmatrix} 924.903 & 0 & 240.950 \\ 0 & 1369.218 & 238.553 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.480$
Minimisation with 4 parameters : $\beta = \delta_0 \alpha$	
$\mathbf{A} = \begin{bmatrix} 1520.311 & -422.906 & 358.637 \\ 0 & 1452.135 & -181.767 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 0.955$	$\mathbf{A} = \begin{bmatrix} 1457.474 & -1074.019 & -848.312 \\ 0 & 1392.116 & -1506.356 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 0.955$
Minimisation with 3 parameters : $\gamma = 0$ and $\beta = \delta_0 \alpha$	
$\mathbf{A} = \begin{bmatrix} 0.322 & 0 & 557.168 \\ 0 & 0.307 & -181.767 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 0.955$	$\mathbf{A} = \begin{bmatrix} 564.535 & 0 & 217.345 \\ 0 & 539.219 & -1506.356 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 0.955$

Table 10: **Synthetic sequence 2** : intrinsic parameters (100 points - Uniform noise, amplitude 0.1).

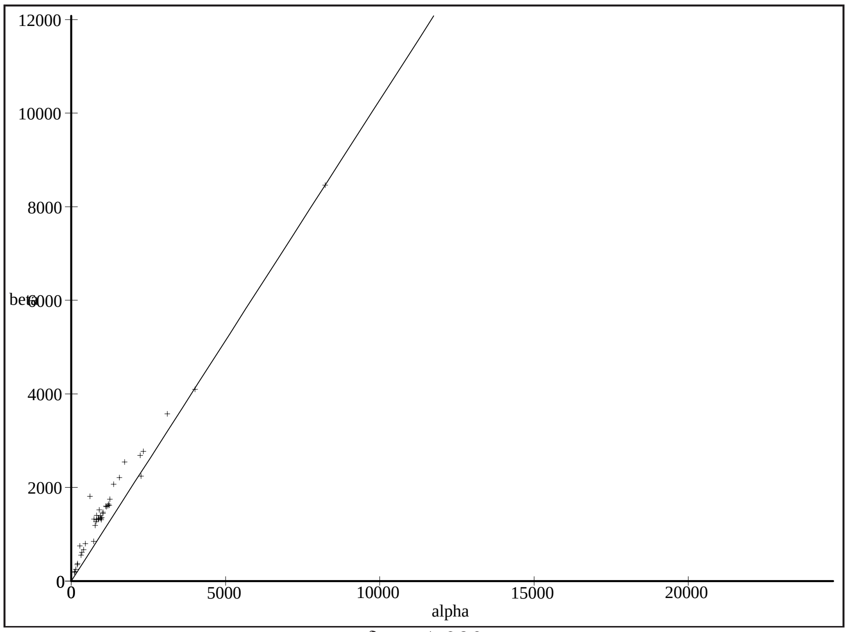
Initialisation	
	
$\delta_0 = 1.028$	
Median	**Exact value**
$\mathbf{A} = \begin{bmatrix} 944.889 & 0 & 256 \\ 0 & 1372.645 & 256 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.453$	$\mathbf{A} = \begin{bmatrix} 914.590 & -0.009 & 249.055 \\ 0 & 1352.105 & 258.933 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.478$
Minimisation with 5 parameters	
$\mathbf{A} = \begin{bmatrix} 1163.564 & 3.824 & 132.135 \\ 0 & 1683.721 & 4.170 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.447$	$\mathbf{A} = \begin{bmatrix} 1162.746 & 3.578 & 132.452 \\ 0 & 1682.509 & 4.669 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.447$
Minimisation with 4 parameters : $\gamma = 0$	
$\mathbf{A} = \begin{bmatrix} 1171.111 & 0 & 126.392 \\ 0 & 1688.727 & -11.081 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.442$	$\mathbf{A} = \begin{bmatrix} 1162.602 & 0 & 131.542 \\ 0 & 1678.165 & -0.387 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.443$
Minimisation with 4 parameters : $\beta = \delta_0 \alpha$	
$\mathbf{A} = \begin{bmatrix} 1872.088 & -434.474 & -341.271 \\ 0 & 1924.877 & -1110.558 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.028$	$\mathbf{A} = \begin{bmatrix} 1872.088 & -434.474 & -341.271 \\ 0 & 1924.877 & -1110.558 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.028$
Minimisation with 3 parameters : $\gamma = 0$ et $\beta = \delta_0 \alpha$	
$\mathbf{A} = \begin{bmatrix} 2436.116 & 0 & -227.784 \\ 0 & 2504.809 & -1110.558 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.028$	$\mathbf{A} = \begin{bmatrix} 2436.177 & 0 & -228.198 \\ 0 & 2504.872 & -1110.558 \\ 0 & 0 & 1 \end{bmatrix}$ $\delta = 1.028$

Table 11: **Synthetic sequence 2** : intrinsic parameters (100 points - uniform noise, amplitude 1.0).

	$\Delta\theta_{\mathbf{r}}$		$\Delta\theta$		$\Delta\theta_{\mathbf{t}}$	
Noise	mean	sd	mean	sd	mean	sd
0.1	0.0732248	0.060809	0.998668	0.235307	0.151225	0.0714225
0.5	0.0794234	0.0610593	1.0458	0.169522	0.195826	0.192501
1.0	0.0849346	0.062585	0.973815	0.223579	0.203342	0.215188

Table 12: **Synthetic sequence 2** : extrinsic parameters (9 displacements - $\Delta\theta_{\mathbf{r}}$, error on the direction of the axis of rotation (radian) - $\Delta\theta$, error on the angle of rotation (radian) - $\Delta\theta_{\mathbf{t}}$, errorr on the direction of translation (radian)).

A real sequence

Figure 4: **Sequence 3** : 6 out of 10 views of the Arcades square in Valbonne (1, 2, 4, 6, 8, 10).

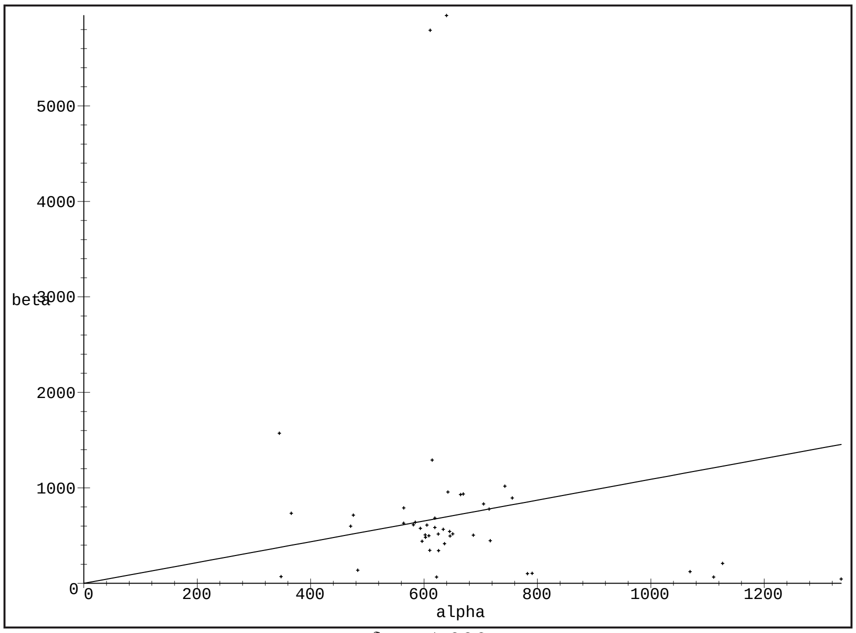
Initialisation

$\delta_0 = 1.089$
Median
$\mathbf{A} = \begin{bmatrix} 625.061 & 0 & 384 \\ 0 & 564.979 & 256 \\ 0 & 0 & 1 \end{bmatrix} \quad \delta = 0.904$
Minimisation with 5 parameters
$\mathbf{A} = \begin{bmatrix} 667.732 & 9.832 & 415.015 \\ 0 & 717.939 & 328.047 \\ 0 & 0 & 1 \end{bmatrix} \quad \delta = 1.075$
Minimisation with 4 parameters : $\gamma = 0$
$\mathbf{A} = \begin{bmatrix} 670.747 & 0 & 416.072 \\ 0 & 729.394 & 334.974 \\ 0 & 0 & 1 \end{bmatrix} \quad \delta = 1.087$
Minimisation with 4 parameters : $\beta = \delta_0 \alpha$
$\mathbf{A} = \begin{bmatrix} 666.705 & 9.897 & 414.542 \\ 0 & 726.272 & 327.606 \\ 0 & 0 & 1 \end{bmatrix} \quad \delta = 1.089$
Minimisation with 3 parameters : $\gamma = 0$ et $\beta = \delta_0 \alpha$
$\mathbf{A} = \begin{bmatrix} 670.566 & 0 & 415.958 \\ 0 & 730.478 & 327.606 \\ 0 & 0 & 1 \end{bmatrix} \quad \delta = 1.089$

Table 13: **Sequence 3** : intrinsic parameters (10 views).

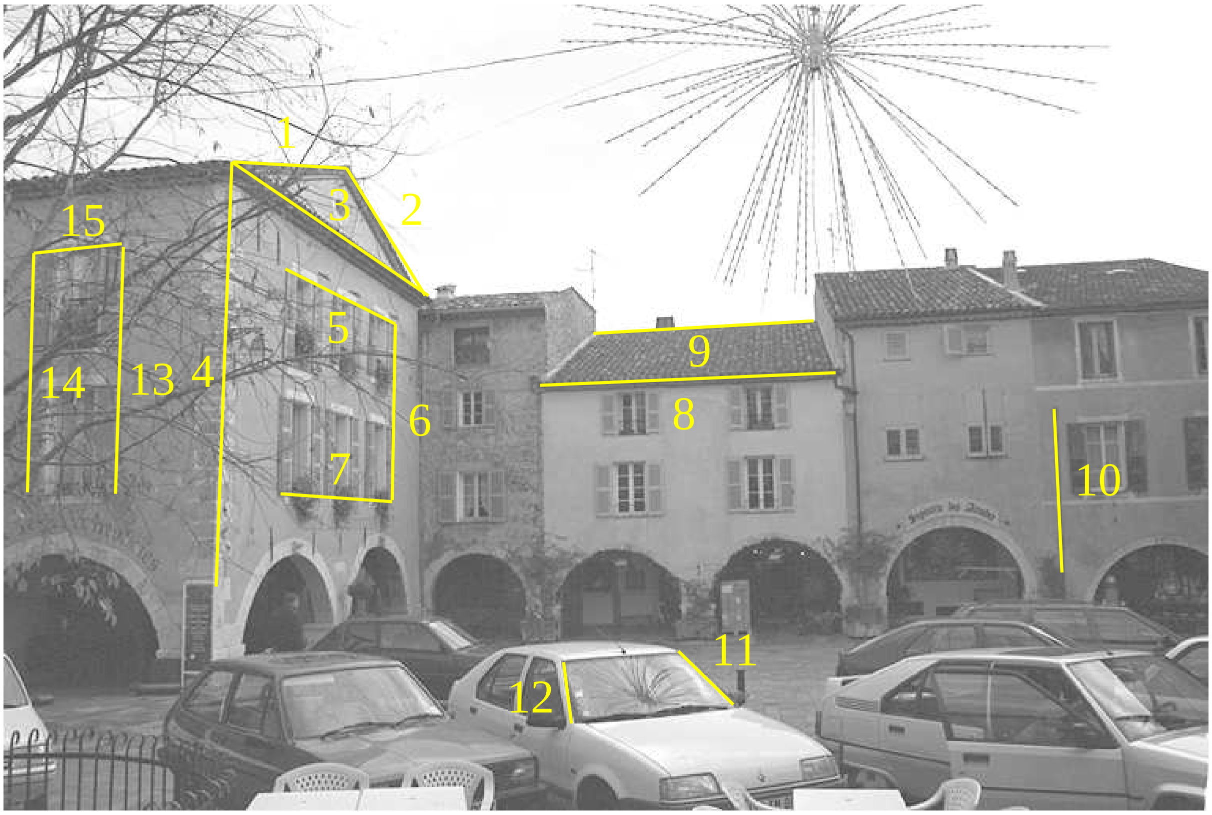


Figure 5: **Sequence 3** : the segments used for the measurements in the next table.

Angle	computed	real	Ratio	computed	real
$\cos(3,4)$	0.031	0	1/2	1.04	1
$\cos(4,8)$	0.105	0	5/7	0.983	1
$\cos(8,10)$	0.032	0	11/12	0.373	1
$\cos(5,6)$	0.031	0	13/14	0.994	1
$\cos(13,15)$	0.07	0			

Table 14: **Sequence 3** : measurements obtained from the estimated intrinsic parameters with $\theta = \pi/2$.