Published in the First International
Symposium on Experimental Robotics
Montreal, Canada
June, 1989
LNCINS, Springer-Verlag, 1990

Determination of Manipulator Contact Information from Joint Torque Measurements

Brian S. Eberman
J. Kenneth Salisbury

MIT Artificial Intelligence Laboratory
Cambridge MA 02139

Abstract

Measurements of the net forces acting on a structure can provide information necessary to determine the location of contacts through which the external forces act, under appropriate assumptions. One practical application of this idea resulted in the contact-resolving force-sensing fingertips developed by Salisbury and Brock. These fingertip sensors employ a 6-axis force sensor under a fingertip shell to measure the net applied force. It has been demonstrated that this type of sensor supplies enough information to determine the location and force components of contacts which transmit pure forces. Bicchi recently extended this analysis to permit the same sensor to also measure a moment exerted about the contact normal.

In a similar manner, we can sense contacts with the last link of a robot arm by measurement of the joint torques alone. Because the arm acts as a force sensor with varying geometry, the conditioning of the solution can vary enormously, resulting in configuration dependent accuracy. This paper describes an approach to joint-torque-based contact sensing for single contacts on the last link of a manipulator and discusses factors affecting the accuracy of the solution. Experimental results with the approach are reported for a four degree-degree-of-freedom arm (the MIT-WAM arm).

1. Introduction

By definition, manipulation requires contact between the robot and the object being manipulated. Much of current robotic practice relies on precise prepositioning of objects to manipulated, and uses only joint position information to guide the robot to the object position. As we move toward less structured manipulation, robots become more reliant on sensory (and ultimately perceptual) feedback. In many non-contact situations vision can be used to advantage. However, when contact is occurring (such as in grasping and pushing operations) more precise and intrinsically mechanical information is required about the contact. Traditionally, tactile sensors have been considered for this type of contact sensing. Such devices employ surface mounted arrays of force sensitive elements which can be used to reveal contact locations, shapes and normal force components [Siegel 86].

An alternative approach, which we consider here, relies on the use of force (or torque) information to reveal the contact location and force components. Previous work in this type of contact sensing dealt with static force sensing structures which employed strain gauges to measure the net force acting on a finger or link [Salisbury 84, Brock 87, Bicchi 89]. Here, we will address the use of joint torque information as a means for determining where contacts occur on a manipulator's surface. This is similar to our earlier strain-gauge-based approaches except that now the structure of the "sensor" may vary as the manipulator's joint positions vary. [Gordon and Townsend, 89] addressed the problem of locating contacts on the last link of a planar 2-link mechanism and compared the accuracy possible in this force based approach with that of a tactile sensor based approach.

The advantage of this joint-torque-based approach for determining contacts is that it may be employed in any manipulator (or hand) that has the ability to measure joint torques. It does not rely on complex tactile sensor arrays to determine contact locations and it gives more complete information about the actual forces transmitted through a contact. There are limitations to using a force-based approach to contact sensing. The actual shape and pressure distribution of the contact cannot be determined and multiple contacts can not be distinguished.

2. Elementary Considerations

If the forces which act upon a manipulation mechanism sum together so as to yield no net acceleration of the mechanism, it is said to be in a state of static equilibrium. These forces may arise from actuators, body forces (i.e. weight of links) and contacts with the environment.

Although the net forces on the bodies of a system in static equilibrium will sum to zero, there will be internal forces which are necessarily nonzero (i.e. structural stresses, contact forces etc.) If measurements of some of these forces can be made (i.e. by force sensing elements in the structure or actuators) then they can be checked for consistency with the forces expected for a particular configuration of the mechanism and used to deduce information about contact.

As a trivial example, consider a single link supported by an actuator from which torque and angle measurements are available. Assuming we know the link's mass, at any position we may predict the torque necessary to support the link against gravity in the absence of contact. If the joint is servoed to a particular position and the actuator torque deviates appreciably from our model's prediction, then we can deduce that contact of some sort is occurring on the link. Even in this simple case, the contact induced torque may arise from a wide variety of contact situations and without further sensing or contact constraint, details of the exact nature of the contact cannot be inferred. Furthermore, the contact forces would be unobservable if they acted only along an axis parallel to the joint axis and the contact would not be detected.

This simple example points out two important aspects of force-based contact sensing: the number of independent force measurements must be equal to or greater than the number of variables which affect the contact force and the space which the force sensors monitor must fully span the space of forces which can be generated by the contacts. In other words, to solve for the attributes of contact from force measurements we must have enough sensors and they must be placed correctly. The number of sensors required will depend upon the number of unknowns necessary to characterize the problem. In the following analysis we assume that only single contacts occur and are to be sensed. While multiple contacts may be perceived with a sufficient number of sensors, our analysis will address the more fundamental case of single contacts.

3. Contact Characterization and Constraint

A single contact may be characterized by two attributes: the force and torque transmitted through the contact and the location and shape of the contact area. In this paper we shall restrict our selves to the important class of contacts which may be approximated by a single point. With this restriction we can characterize a contact occurring on the link of a manipulator by the *contact wrench*, $\mathbf{W}$, and the contact position, $\mathbf{r}$.

$\mathbf{W}$ is a 6-vector of the three force and three moments which are transmitted through the contact. In the contact frame of reference we represent this wrench as ${}^{C}W = (\mathbf{f}, \mathbf{m})$, with $\mathbf{f}$ and $\mathbf{m}$ equal to the force and moment contact applied through the contact, respectively. $\mathbf{r}$ is the three element vector giving the contact's position. There will be, in general, nine unknowns which we must solve for in order to characterize a contact. Depending upon the type of contact, we must find an appropriate combination of constraints and measurements to apply in order to solve for these nine quantities.

The shape of the link upon which the contact is to be detected establishes the first set of constraints. If the contact occurs on the surface of a link of whose surface equation is given by

$$S(\mathbf{r}) = 0, \tag{1}$$

then the number of constraints imposed will depend upon the type of surface. If S is a general surface then one constraint is imposed upon $\mathbf{r}$. If S is line, then two constraints are imposed upon $\mathbf{r}$. If S is a point then $\mathbf{r}$ is completely determined by the three constraints imposed upon it.

The second set of constraints arises from restrictions on the applied forces due to friction and contact geometry. For the purpose of discussion, we assume that a measurement of the net force and torque acting on the link, expressed in a base coordinate frame O, is given by ${}^{\mathbf{0}}\mathbf{W} = (\mathbf{F}, \mathbf{M})$ where $\mathbf{F}$ and $\mathbf{M}$ are actual measured quantities. We assume $\mathbf{r}$ is also expressed in frame 0.

If the contact is a *point contact with friction*, the contact wrench must be a pure force (i.e. a zero-pitch wrench) implying that

$$\mathbf{m} \cdot \mathbf{f} = 0. \tag{2}$$

Furthermore, this contact permits no moment to be exerted in the contact frame of reference, implying that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}. \tag{3}$$

If the contact is a *frictionless point contact* then, in addition to the constraints imposed by eqn(2) and eqn(3) above, the contact force must be normal to the surface at the point of contact:

$$\mathbf{f} \times \nabla S(\mathbf{r}) = 0. \tag{4}$$

Finally, a *soft finger contact* may exert an arbitrary force through the contact and a moment normal to the contact plane such that:

$$\mathbf{m} \times \nabla S(\mathbf{r}) = 0. \tag{5}$$

Eqn(2) imposes one constraint on the values of $\mathbf{f}$ and $\mathbf{m}$. Eqn(3) imposes two constraints on the value of $\mathbf{r}$ as can be see by solving it for $\mathbf{r} = \frac{\mathbf{F}\times\mathbf{M}}{\mathbf{F}\cdot\mathbf{F}} + \lambda\frac{\mathbf{F}}{||\mathbf{F}||}$ leaving the free variable (λ). Eqn(4) and eqn(5) both each indicate that a vector is normal to a surface. The constraints imposed by these conditions will depend upon the type of the surface. If $S(\mathbf{r})$ is a general surface then eqn(4) and eqn(5) each impose two constraints. If $S(\mathbf{r})$ is a line, then the equations only impose one constraint each, since the surface normal defined only to lie in a plane normal to the line's direction. If $S(\mathbf{r})$ is a point then eqns(4) and (5) impose no additional constraint, since the surface normal at a point is indeterminant.

If we subtract the number of constraints imposed by link shape and contact type from the original number of variables (nine) we may determine the minimum number of force sensors which must be employed to characterize a particular contact. For example if a point contact with friction occurs on a general body, the constraints are as follows: contact on general body - one constraint, zero pitch wrench - one constraint, no moment at contact - two constraints. The minimum number of sensors is $9-1-1-2 = 5$. Although this is theoretically the minimum, only in special cases can 5 sensors be arranged so that they always span the forces exertable from all contacts on a surface.

Later in this paper will consider the case of a point contact with friction occurring on a link which is slender enough to be considered to be a line. The constraints will be: contact on a line - two constraints, zero-pitch wrench - one constraint, no moment at contact - two constraints. Thus the minimum number of sensors is $9 - 2 - 1 - 2 = 4$. In this case, our sensors will be the joint torque measurements taken from the four joints through which forces applied to the link must pass.

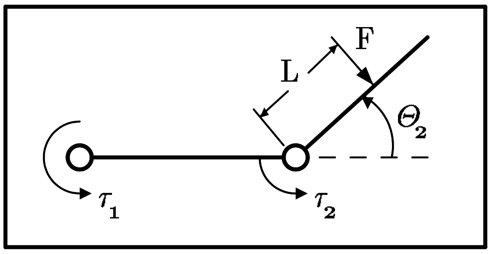

Figure 1. Planar 2 DOF Mechanism contact determination.

4. Link Contact Problem

The basic problem is illustrated by considering a frictionless point contact on a line occurring in a plane. In this case there are seven constraints, six from the frictionless contact on a line and an additional one from the planar constraint. We consider in Figure 1, the basic problem of determining contact locations on a link from joint torque information where a force f is applied to the last link at a distance L from the last joint. If we assume that the joint angles are known then the vector of joint torques τ may be written as:

$$\tau = (l + eL)f \tag{6}$$

where l is a matrix composed of elements dependent upon the known link lengths and joint angles and e is a matrix composed of elements dependent only on the joint angles. For the simple planar case shown in Figure 1, if we assume the contact force f to be transmitted through a frictionless contact, then f is the scalar magnitude of the contact force and $l = (l_1\, cos(\theta_2)\;\; 0)^T$ and $e = (1\;\; 1)^T$. Assuming f and L are to be determined from a measurement of $\tau = (\tau_1\;\; \tau_2)^T$ we can solve eqn (6) to find

$$f = \frac{\tau_1 - \tau_2}{l_1 cos(\theta_2)} \tag{7}$$

and

$$L = \frac{\tau_2 l_1 cos(\theta_2)}{\tau_1 - \tau_2} \tag{8}$$

where θ_1 is assumed to be 0 without loss of generality. An important problem inherent in this approach occurs when $\theta_2 = \pm\pi/2$. In this case the two equations implied by eqn 6 cease to be independent and no solution is possible. Even in the neighborhood of this singularity the equations become ill-conditioned and sensitive to error in torque measurement. This illustrates a problem inherent in solely using joint torque measurement for determining a contact's location and force components. The configuration dependent conditioning renders parts of the workspace inappropriate for force based contact sensing.

One possible remedy to this situation is to utilize a tactile sensor to determine the contact's location and use the joint torque data to determine the magnitude of the contact force(s). With L known, eqn (6) reduces to a pair of linear equations which are over-determined. While either the expression for τ_1 or τ_2 alone could be used to find f, a more sensible approach, assuming equally distributed Gaussian zero mean noise on the torques, would be to use a least squares solution to eqn (6) so that

$$f = (\mathcal{L}_T \mathcal{L})^{-1} \mathcal{L} \tau \tag{9}$$

where $\mathcal{L} = (l + eL)$. This approach would permit us to take advantage of the tactile sensor, which often has poor force resolution, to find the contact location while the joint torque data is used to determine the contact force.

5. Four degree-of-freedom Arm Solution

For the MIT-WAM arm, we will consider single point contacts with friction occurring along the last link and, as a simplifying assumption, treat the last link as a line. This assumption will introduce errors to the extent that the contact force's line of action does not pass through the last link's central axis.

For a point contact with friction, five constraints are imposed on the nine parameters. We can write six independent equations for the components of the contact wrench, four from the magnitudes of the torques measured at each joint, the fifth from the constraint that the line of action of the force must intersect the last link, and the last constraint from the requirement that the wrench be the resultant of a pure force.

Because the arm can assume a wide variety of configurations, it is possible to find arm postures and wrenches where the equations are not independent and solving for accurate contact information is impossible. We detail these conditions and define a measure of local conditioning for the problem. Using the conditioning result we will present experimental results for a well conditioned configuration.

5.1. Kinematics and Plücker Coordinates

The kinematics of the MIT-WAM are shown in Figure 2. In this system, ${}^i z_j$ is the vector of the j^{th} joint axes expressed in the i^{th} coordinate system. The base system is the 0 frame. The Denavit-Hartenberg parameters for the arm, with this set of frames, are listed in the following table:

coordinate frame number i	link twist (radians) α_{i-1}	link length a_{i-1}	link offset d_i	joint angle θ
1	0	0	0	θ_1
2	$\pi/2$	0	0	θ_2
3	$-\pi/2$	0	L_3	θ_3
4	$\pi/2$	A_3	0	θ_4
5	$-\pi/2$	$-A_4$	0	0

L_3 is the inner link length of 22 inches. The outer link length is 17.5 inches. A_3 and A_4 are the offsets in the manipulator, 1.6 inches and 1.1 inches.

A line is designated by $L = (\hat{L}, \tilde{L})$ where $\hat{L}$ is the normalized direction of the line, and $\tilde{L}$ is the moment of the normalized direction about the origin of the coordinate system.

The virtual product of two lines, $\otimes$, (or a line and a wrench) is

$$L_1 \otimes L_2 = \hat{L}_1 \cdot \tilde{L}_2 + \tilde{L}_1 \cdot \hat{L}_2$$

The virtual product of two lines goes to zero when two lines intersect (including intersections at infinity). The line product of a line and a wrench gives the magnitude of the torque about the line.

5.2. Contact Solution

We consider the problem of determining the line of action and magnitude of a pure force applied along the line through the center of the last link of the MIT-WAM. This force can be written as a general wrench ${}^0W$ in the base coordinate system. Four equations of the wrench can be found by taking the virtual product of the wrench with the the lines along each of the four joint axes (${}^0L_i$: $i \,\epsilon\, 1, 2, 3, 4$) which is equal to the joint torque.

The constraint that the force act along the line of the last link produces the constraint that the virtual product of the wrench with the line through the last link L_5 must equal 0. The final sixth constraint comes from the constraint that the dot product of $\hat{W}$ with $\tilde{W}$ is zero because the wrench is a pure force. The six equations for the six unknown terms of the wrench are:

$$\begin{aligned} \tau_1 &= L_1 \otimes W \\ \tau_2 &= L_2 \otimes W \\ \tau_3 &= L_3 \otimes W \\ \tau_4 &= L_4 \otimes W \\ 0 &= L_5 \otimes W \\ 0 &= \hat{W} \cdot \tilde{W}. \end{aligned} \tag{10}$$

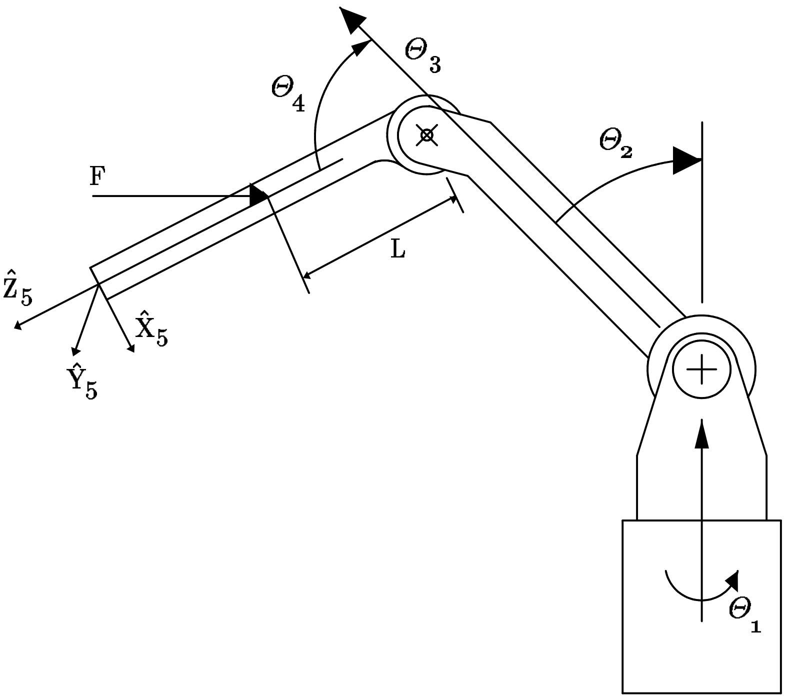

Figure 2. MIT-WAM Kinematics

This set of equations can be simplified by expanding L_i in terms of its components in the base frame $({}^0\hat{z}_i, {}^0\tilde{z}_i)$. In this frame, $\tilde{z}$ is zero for the first three lines. In other words the first three joint torque measure the moment of the wrench which is the moment of the applied force about the base. Performing the expansion results in:

$$\begin{aligned} \tau_1 &= {}^0\hat{z}_1 \cdot {}^0\tilde{W} \\ \tau_2 &= {}^0\hat{z}_2 \cdot {}^0\tilde{W} \\ \tau_3 &= {}^0\hat{z}_3 \cdot {}^0\tilde{W} \\ \tau_4 &= {}^4\hat{z}_4 \cdot {}^4\tilde{W} + {}^4\tilde{z}_4 \cdot {}^4\hat{W} \\ 0 &= {}^4\hat{z}_5 \cdot {}^4\tilde{W} + {}^4\tilde{z}_5 \cdot {}^4\hat{W} \\ 0 &= {}^4\hat{W} \cdot {}^4\tilde{W}. \end{aligned} \tag{11}$$

The solution for W is then found by a two step procedure. First, the first three equations are solved for ${}^0\tilde{W}$. Then, for increased speed of computation, a rotation is applied to ${}^0\tilde{W}$ to take it to ${}^4\tilde{W}$. Finally, given the value of ${}^4\tilde{W}$, the second set of equations can be solved for ${}^4\hat{W}$. In the implementation of this procedure, we have symbolically inverted the first three equations.

To determine the location of contact, the plane through the origin of the base frame normal to ${}^4\tilde{W}$ is intersected with the line ${}^4L_5$. Because the wrench is a pure force $\tilde{W} = r \times \hat{W}$, where r is the vector to the point of application of the force, the point of contact must lie in the plane normal to $\tilde{W}$. The distance to the contact measured along the line from the origin of the fifth frame is then:

$$d = -({}^4\hat{W} \cdot {}^4O_5)/({}^4\hat{W} \cdot {}^4\hat{z}_5) \tag{12}$$

where ${}^4O_5$ is the vector to the origin of the fifth frame in the fourth coordinate system.

6. Singularities and Conditioning

Because of the kinematics of the manipulator, the general nonlinear form of the contact problem simplifies to a two step linear problem. We now examine the conditions under which this procedure becomes singular.

The first step, solving for $\tilde{W}$, involves the inversion of a 3×3 matrix consisting of the unit directions of the first three joints. This matrix becomes singular when $\theta_2 = 0$. The conditioning for this matrix is determined by the ratio of the maximum to minimum singular values which is found to be:

$$\frac{1 + |cos(\theta_2)|}{1 - |cos(\theta_2)|} \tag{13}$$

For good performance of the calculation, θ_2 must be near $\pi/2$.

The second step, solving for $\hat{W}$, involves the inversion of a 3×3 matrix involving $\tilde{z}_4$ and $\tilde{z}_5$ in addition to $\tilde{W}$. To determine singularity we examine when these vectors become linearly dependent. First, by taking the dot product of $\tilde{z}_4$ with

$\tilde{z}_5$ we find that they are always perpendicular and so form a basis against which $\tilde{W}$ can be evaluated.

The non-zero length links of the robot always form a plane P. $\hat{z}_4$ is always perpendicular to this plane and $\hat{z}_5$, and all other vectors to points on the robot, lie within P. The applied force, $\hat{W}$, can be decomposed into a vector in the plane of the robot ($\hat{W}_P$) and a force perpendicular to the plane $\hat{W}_N$. Because $\tilde{W}_P$ must be perpendicular to $\hat{W}_P$ and the vector from the origin to the point of contact, which lies in the plane of the robot, $\tilde{W}_P$ must be perpendicular to P or in the direction of $\tilde{z}_5$. Therefore, $\tilde{W}_P$ is always linearly dependent on $\tilde{z}_5$.

Similarly, $\tilde{W}_N$ can be shown to be perpendicular to $\tilde{z}_5$ and so can only depend on $\tilde{z}_4$. In order for $\tilde{W}_N$ to be linearly dependent on $\tilde{z}_4$, the contact location must be in the direction ${}^0O_4$. By intersecting this line with the last link, we find the unique contact location which will cause linear dependence to be:

$$L = \frac{A_3 A_4 S_4 - A_4 L_3 C_4}{L_3 S_4 + A_3 C_4}. \tag{14}$$

In summary, in order for there to be a unique solution the force must have a component perpendicular to the plane of the robot, the force must be applied at a location other than the location given by eqn (14), and θ_2 cannot equal 0. Away from these conditions, a unique solution exists. However, the conditioning of the problem may still be poor. To determine the conditioning, and select experimental locations, we examined the condition number of the differential relationship between the applied wrench and the torques. For a given wrench, the differential relationship is a 6×6 matrix of constants and so a condition number can be determined.

7. Experimental Results

We present results for this method at a well conditioned location. All experiments were done in a static condition.

The joint torques are inferred from the errors in positioning. We assume that the torques acting on the robot consist of the gravity forces, motor ripple, friction, and applied forces. The controller applies a feedforward correction term for both the gravity forces and the ripple. A switching controller is used to decrease the effects of friction. The remaining torque should be due only to the applied forces. Unfortunately, we have found that there remains a significant variation in the torques due to ripple which depends upon the magnitude of the total torque. Since, the gravity torque is greatest at locations with θ_2 near $\pi/2$, causing an increase in the ripple magnitude, this effects seriously degrades the performance of the calculation at the well conditioned locations.

For the well conditioned experiment, we selected joint angles $(0, -1.57, -1.57, -1.57)$. At this location we experimented with varying the contact location along the second link with a constant force of 2 lb in the ${}^5\hat{y}$ direction (see figure 2). At this location we found numerically that the condition number increased by 0.9 for every inch that the contact location was increased from a starting value of 52.8 at 5 inches. Figure 3 shows the variation in contact location estimate with actual contact location and Figure 4 shows the variation in contact force estimate with contact location.

To perform the experiment, we hung a 2lb weight on the last link at the experimental locations. The robot was servoed back to the original location using stiffness and integral control. The torque computed from the errors in positioning and the integral of the position error were used to determine the joint torque. Errors in positioning and the increase in motor torque ripple magnitude with the mean motor torque contribute to the errors.

8. Discussion and Future Improvements

Manipulation requires the sensing and perception of contact information between the robot and the workspace. An internal force sensor can determine the location and magnitude of a contact for certain geometries, providing a robust and simple technique for determining this important information. For the special case of the manipulator joint torques, we have a time varying geometry and thus a time varying conditioning to the problem.

The changes in conditioning with position, and the singularity of the problem to forces in the plane of the robot, imposes restrictions on the utility of this approach for the MIT-WAM with the current configuration of torque sensors. By adding additional torque sensing to the manipulator so as to minimize the condition number over the workspace of positions and forces, we believe the performance of this idea can be greatly improved.

Our procedure also suffers from the inference of motor torques from position errors. Motor ripple and errors, deadband in the motor controllers, errors in calibration of the torque-voltage command curves, and gravity torque calibration errors all

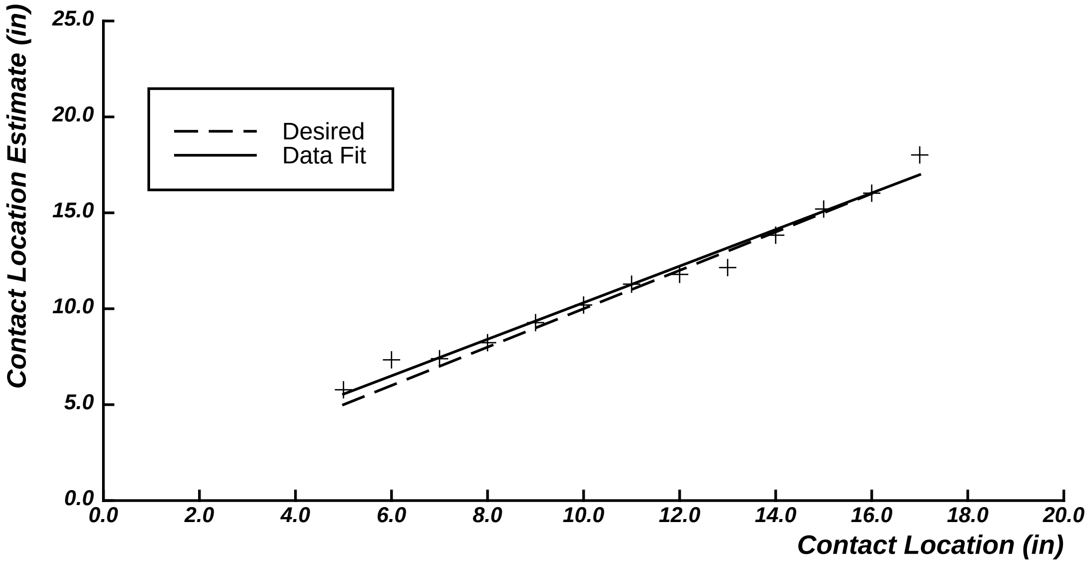

Figure 3. Variation in contact location estimate with actual location

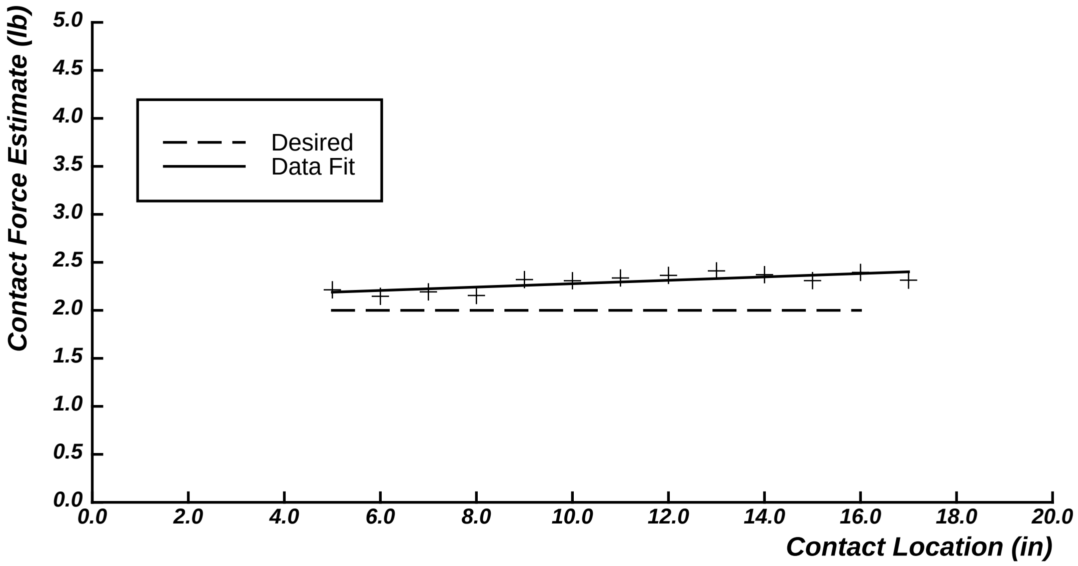

Figure 4. Variation in contact force estimate with contact location

contribute to errors in the contact torque measurement. In addition, for higher speed motions, the acceleration of the robot links must be subtracted from the observed torques in order to determine the contact torques creating additional sources of error. Direct sensing and control of motor torque would reduce these problems.

Array tactile sensing suffers from the number of signals that must be analyzed and from an often poor resolution of force. However, a low density tactile array (with only a few elements per square inch) would provide sufficient resolution of location for many tasks. Given a value for the contact location, from an array tactile sensor, determination of the the contact forces from the joint torques becomes a linear problem. This approach remains to be investigated.

9. Acknowledgments

The authors would like to gratefully acknowledge the financial assistance of the Office of Naval Research for support in this research under ONR contracts N00014-86-K-0685 and N00015-85-K-0124.

References

[1] Bicchi, A. “Strumenti e Metodi per il Controllo di Mani per Robot,” Phd. Thesis, University of Bologna, 1989.

[2] Brock, D.L., "Enhancing the Dexterity of a Robot Hand Using Controlled Slip," Master's Thesis, Department of Mechanical Engineering, MIT, June, 1987. MIT AI-TR 992.

[3] Eberman, B.S., "Whole Arm Manipulation: Kinematics and Control," Master's Thesis, Dept. of Mechanical Engineering, MIT, Cambridge, MA, January 1989.

[4] Hunt, K.H.,*Kinematic Geometry of Mechanisms*, Oxford University Press, London, England, 1978.

[5] Salisbury, J.K., Townsend, W.T., Eberman, B.S., DiPietro, D.M., "Preliminary Design of a Whole-Arm Manipulation System (WAMS)," Proc. of 1988 IEEE International Conf. on Robotics and Automation, Philadelphia, PA, April 1988.

[6] Salisbury, J.K. "Whole-Arm Manipulation," Proc. of the 4th International Symposium on Robotics Research, Santa Cruz, CA August 1987. Published by the MIT Press, Cambridge MA.

[7] Salisbury, J.K., "Interpretation of Contact Geometries from Force Measurements," *proc. 1st International Symposium on Robotics Research,* Bretton Woods, NH, September 1984, published by the MIT Press, Cambridge MA.

[8] Shimano, B.E., "The Kinematic Design and Force Control of Computer Controlled Manipulators," Ph.D. thesis, Department of Computer Science, Stanford University, March 1978.

[9] Siegel, D.M., "Contact sensors for dexterous robotic hands," Technical Report 900, MIT Artificial Intelligence Laboratory, Cambridge, MA, August 1986.

[10] Townsend, W.T., "The Effect of Transmission Design on Force-Controlled Manipulator Performance," Ph.D. thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology, April 1988.