Relation Between Brockett's Algorithm and GHA

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Brockett's Algorithm for finding principal components can be written

$$
\begin{aligned}
y &= Wx & (1)\\
\Delta W &= \gamma(Nyx^T - yy^T NW) & (2)
\end{aligned}
$$

where W is the matrix of feedforward weights, γ is the learning rate, and N is a diagonal matrix with strictly decreasing entries.

Sanger's Generalized Hebbian Algorithm (GHA) can be written

$$\Delta W = \gamma(yx - \mathrm{LT}[yy^T]W) \tag{3}$$

where LT[] sets the above diagonal elements of its matrix argument to zero.

To see the relation between the two algorithms, multiply both sides of equation 2 by N^{-1} to obtain

$$N^{-1}\Delta W = \gamma(yx^T - N^{-1}yy^T NW) \tag{4}$$

and consider the second term $A = N^{-1}yy^T N$. This is a matrix with elements given by

$$a_{ij} = \left(\frac{n_j}{n_i}\right) y_i y_j.$$

Since $n_0 > n_1 > \ldots$, we see that

$$a_{ij} = \begin{cases} y_i^2 & i = j \\ < y_i y_j & i < j \\ > y_i y_j & i > j \end{cases}$$

which means that the elements of the matrix A are less than the elements of yy^T above the diagonal, greater below the diagonal, and equal along the diagonal.

So GHA can be thought of as a rectified form of Brockett's algorithm, in which the matrix A is given by

$$a_{ij} = \begin{cases} y_i^2 & i = j \\ 0 & i < j \\ y_i y_j & i > j \end{cases}$$

and it is not surprising that both algorithms converge to the same solutions at approximately the same rate.