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6.313 Contemporary Computer Design, Fall 1995

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Problem Set #1 Solutions, Sept 20, 1995

1. (a) $\mathcal{R} = \rho/\pi r^2 = 0.07\Omega/m = 350k\Omega/wire$

$\mathcal{C} = K\epsilon_0 w/h \approx K\epsilon_0(d + \frac{3}{4}h)/h$
To go from two parallel conductors to a wire above a plane, insert a ground plane between the two conductors. The $\vec{E}$ field is perpendicular to this plane and the total capacitance is 1/2 the capacitance of one wire to the ground plane.
$(1/C_T = 1/C + 1/C \Rightarrow C_T = C/2)$
So $\mathcal{C} = K\epsilon_0 \frac{1}{2}(\frac{d+\frac{3}{4}s}{s}) \approx 3/8K\epsilon_0 = 3.3pF/m$

A more exact treatment leads to the formula $\mathcal{C} = \frac{\pi\epsilon_0 K}{cosh^{-1}(s/d)}$
We find that $cosh^{-1}$ of a large number is of order 10, and for s=1m and d=0.56mm, $\mathcal{C} = 3.4pF/m$ so our approximation is very good.

To compute $\mathcal{L}$ we use the fact that $1/\mathcal{LC} = c^2/MK \Rightarrow \mathcal{L} = MK/c^2\mathcal{C}$
so $\mathcal{L} = 3.4\mu H/m$
A better analysis leads to $\mathcal{L} \approx M\mu_0/\pi ln(\frac{s}{d/2} - 1) = 3.3\mu H/m$
An even better analysis leads to $\mathcal{L} = M\mu_0/\pi cosh^{-1}(s/d) = 3.3\mu H/m$ so again our approximations are good.

(b) To choose a model for the telegraph line, compare Z_0 to R (not $\mathcal{R}$). $Z_0 = \sqrt{\mathcal{L}/\mathcal{C}} = 1015\Omega$
$R = (0.07\Omega/m)(5000km) = 350,000\Omega$
Clearly the signal speed is (would be) dominated by RC effects. Unfortunately, because of the 350kΩ resistance of each wire, the solenoid will never see a 50V drop across it, and the signaling system just will not work.

(c)

(d) Clearly both voltage and current signaling are ineffective means to transmit information over this distance. A much larger voltage source and much more sensitive receivers could work, but the requirements are difficult to achieve. A better solution could be to decrease the resistance of the wire. A wire with a 1cm diameter would reduce the resistance to approximately 1100Ω. Restringing 5000km of wire is less than ideal however.
An even better solution would be to buffer the wires periodically. Given a repeater setup identical to the original (5kΩ solenoid, 100V supply) buffers would be required every 35km for the system to merely work. To work at a reasonable speed requires even more frequent repeaters.
A less obvious solution is to use current signaling and use "load coils" (large series inductances) placed along the route. This extra inductance raises the characteristic impedance, making the line behave as a transmission line.

2. (a) $E = \frac{1}{2}CV^2 = 900\mu J$

(b) $\mathcal{R} = (4\Omega/\Box)(250\mu m/5\mu m) = 200\Omega$

(c) $C = CJ(250\mu m \times 5\mu m) + 2 \times CJSW(250\mu m + 5\mu m) = 780fF$

(d) If we assume that internal structures will restore the level of the signal, we can accept a signal which is 63% of the input signal. Then switching speed is equal to the RC delay. The RC delay through the device is 150ps, for a maximum frequency of 6GHz.

(e) $G = (4 \times 10^{10})(250\mu m \times 5\mu m) = 50S$
$R = 1/G = 0.02\Omega$

(f) Resistance per unit length is the total resistance of the N+ strip divided by the length of the strip. Conductance per unit length is the conductance between the N+ strip and the substrate.
$\mathcal{R} = 200\Omega/250\mu m = 0.8\Omega/\mu m$ $\mathcal{G} = 50S/250\mu m = 200kS/m = 5\mu\Omega/m$

(g) As in the notes, we assign voltages V_-, V_0, and V_+, to the left, center, and right nodes respectively, and write charge conservation for node V_0,

$$\frac{(V_- - V_0)}{\mathcal{R}\delta x} = \frac{(V_0 - V_+)}{\mathcal{R}\delta x} + \mathcal{G}\delta x V_0. \tag{1}$$

Taking the limit as δx approaches zero, we arrive at the governing equation for the resistive conductive line,

$$\frac{\partial^2 V}{\partial x^2} = \mathcal{R}\mathcal{G}V. \tag{2}$$

(h) Solutions to this equation are of the form

$$V(x) = ke^{-\sqrt{RG}x} \tag{3}$$

(i) As can be seen from the form of the solution for V(x), voltage drops exponentially with position. The RG network can be viewed as a series of voltage dividers, so the voltage at the end of the network is much lower (hopefully below the breakdown voltage of the diodes) than at the input.

(j) As circuit elements scale, the RC delay through the input devices remains fairly constant, but the internal circuits are running faster. This presents a bottleneck at the input protection devices. The energy which must be dissipated by the protection device does not scale with the technology, and indeed may become even larger. The RC delay through this device therefore may be very significant.

3. (a) This is a tricky problem with multiple solutions. The first assumption which must be made is the size of the contact between the power supply and the board. For a zero size contact, the resistance is infinite and all the voltage is dropped. So we will assume a 1cm contact to the board. To determine the voltage drop across the board we will make a few "simplifying" approximations. First, we will mirror the board about the x and y axes so that the current enters in the center of a board four times the size of the original board, and the total current is raised to 400A. We then approximate the big square as a circle.
We can then compute the resistance of a ring of material as $R_s \frac{dr}{\pi r}$ where r is the distance from the ring to the center of the circle. The voltage drop to the edge of

the ring is:

$$V = I \int_a^b R_s \frac{dr}{\pi r} \tag{4}$$

where a and b are the inner radius of the contact to the power supply and the outer radius of the board, respectively, I is the total current through a ring of resistance (400A), and R_s is the sheet resistance. The value of a must be estimated, and the value of b can be approximated as 25cm. For a=1cm, the voltage at the edge is

$$V = 400A(2.51 \times 10^{-4})/\pi ln(\frac{25cm}{1cm}) = 0.103V, \tag{5}$$

which is 5% of the supply voltage.

Another way to solve this problem is to divide the board into squares and compute the resistance to the opposite corner as a parallel resistor network. Using an 8x8 grid and calculating the total resistance of the parallel network, we can estimate the voltage drop as 50mV. Under these circumstances, a factor of 2 is acceptable.

(b) The capacitance between the power and ground planes is close to an ideal parallel plate capacitor. $C = K\epsilon_0 A/d = 3.9\epsilon_0(20^2 + \frac{3}{4}(125\mu m))/125\mu m = 11nF$

(c)

$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{h}{w}\sqrt{\frac{M}{K}}\sqrt{\frac{\mu_0}{\epsilon_0}} \tag{6}$$

$$w = \frac{h}{Z_0}\sqrt{\frac{M}{K}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{250\mu m}{68\Omega}\sqrt{\frac{1}{3.9}}377\Omega = 701\mu m \tag{7}$$

(d)

$$\nu = \frac{c}{\sqrt{MK}} = 0.5c \tag{8}$$

(e) The inductance of a stripline is the same as that for a microstrip, so for a equal Z_0, the stripline width is half that of the microstrip, or $350\mu m$.

(f) The signal speed is $\frac{1}{\sqrt{\mathcal{LC}}}$ and will therefore be $1/\sqrt{2} \times 0.5c = 0.35c$

(g) The stripline is unaffected by immersion in water since it is shielded on both sides by ground planes. The microstrip capacitance is now dominated by a capacitance between the strip and a conductor far above. The capacitance to a far away conductor is a constant $\frac{3}{4}K\epsilon_0$ and for K=80 (water) the total capacitance is 655pF/m. The inductance is the same, 450nH/n, so $Z_0 = 26\Omega$. The speed of propagation is therefore 0.2c