Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.313 Contemporary Computer Design

Problem Set #2 Solutions, September 27, 1995

1. (a) $v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{c}{\sqrt{MK}} = c/2$
2x/10ns = c/2 $\Rightarrow$ x = 0.75m
The end of the line is a short circuit (zero impedance)

(b) The red device is a 150Ω resistor.

(c) The blue device is a 50Ω resistor in parallel with an inductor. The inductor sees the 50Ω resistor in parallel with the series terminated 50Ω line, or 25Ω. The time constant therefore is $\tau = L/R = L/25\Omega = 2ns \Rightarrow L = 50nH$
The waveform at the end of the line spikes up to 1V at 5ns, then exponentially falls to zero with τ of 2ns.

(d) The gold-plated boxes form a 150Ω resistor in parallel with a 75Ω resistor in series with an inductor. The waveform at A spikes up to 1.5V, so the termination initially looks like a 150Ω resistor, and falls with time to look like a 50Ω resistor, or 150 in parallel with 75. The inductor sees a resistance of 50Ω in parallel with 150Ω in series with 75Ω, for a total of 112.5Ω. $\tau = L/R = L/112.5\Omega = 2ns \Rightarrow L = 225nH$

2. (a) $v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{l}{\sqrt{LC}} = c/3$
$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{L}{C}} = 50\Omega$
Solving, C = 36pF, L = 90nH. (and assume M=1, so K=9)

(b) $\mathcal{L} = 500nH/m \qquad \mathcal{C} = 200pF/m$

(c) $C = \frac{K\epsilon_0(200\mu m + \frac{3}{4}100\mu m)(2cm)}{100\mu m} = 4.4pF$

(d) All the capacitors are in parallel, so the total $C = 6 \times (4.4pF + 8pF) + 36pF = 110.4pF$ so $\mathcal{C} = 110.4pF/18cm = 613pF/m$

(e) The cards are modeled as purely capacitive elements, so L is still 90nH.
$Z_0 = \sqrt{\frac{L}{C}} = 28.5\Omega$ so the lines should be terminated with 28.5Ω resistors.

(f) $v = \frac{1}{\sqrt{\mathcal{L}\mathcal{C}}} = \frac{l}{\sqrt{LC}} = c/5.25$

3. The problem is that the traces going to B and C act to reduce the load resistance seen by the 60Ω line from A. One solution is to remove the trace to B and replace it with a 120Ω twisted pair line and run a twisted pair line to C, both lines terminated by 2 60Ω resistors.

The other solution is to leave the 60Ω trace to B, run two twisted pair lines to C, for a total impedance of 60Ω, with the lines going to B and C both terminated by 60 Ω resistors, and lower the impedance of the line from A by running a pair of twisted pair lines for a total impedance from A of 30Ω. This solution does not require the removal of any traces.